PNNL-15432

The Second Generation Model: Model Description and Theory

Allen A. Fawcett[1]
Ronald D. Sands[2]

October
2005

Prepared for the United States Environmental Protection Agency under Contracts AGRDW89921669 and AGRDW89921964

Joint Global Change Research Institute, College Park, MD
Pacific Northwest National Laboratory
Operated by Battelle for the US Department of Energy

---

[1] U.S. Environmental Protection Agency, 1200 Pennsylvania Ave. NW (6207J), Washington, DC 20460, USA. The views of the authors do not necessarily represent the views of the U.S. Government or the Environmental Protection Agency.

[2] Joint Global Change Research Institute, Pacific Northwest National Laboratory, Battelle, University of Maryland, 8400 Baltimore Avenue, Suite 201, College Park, MD 20740, USA

LEGAL NOTICE

This report was prepared by Battelle Memorial Institute (Batelle) as an account of sponsored research activities. Neither Client nor Battelle nor any person acting on behalf of either:

MAKES ANY WARRANTY OR REPRESENTATION, EXPRESS OR IMPLIED, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, product, or composition disclosed in this report may not infringe privately owned rights; or

Assumes any liabilities with respect to the use of, or for damages resulting from the use of, any information, apparatus, process, or composition disclosed in this report.

Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by Battelle. The views and opinions of authors expressed herein do not necessarily state or reflect those of Battelle.

The Second Generation Model:
Model Description and Theory

1. INTRODUCTION

The purpose of this document is to provide a complete theoretical description of the current structure of the Second Generation Model (SGM). Development of the SGM is an ongoing process, and changes to the structure described in this document are anticipated. The structure of the model described in this document is current as of October 2005.

The SGM was constructed to address the climate problem in general, but more specifically to allow for analysis of alternative climate policies over a time period of five to fifty years. Our focus on climate has influenced model design in many ways including the need for global coverage, selection of a computable general equilibrium (CGE) framework, realistic representation of energy technologies, and capturing important energy system dynamics through capital stock turnover. While this document covers the theoretical structure of the SGM, a companion document (Sands and Fawcett, 2005) covers the current data requirements and parameterization of the SGM.

The model is scaled to represent aggregate behavior of nations and regional aggregations of nations. The model is a recursive dynamic model that solves in five year time steps. It is currently run forward 50 years into the future. The model is therefore appropriate for addressing questions associated with energy-economy-climate policy interactions for economies over a medium-term time horizon. Several model features are worth noting and play an important role in shaping model structure. These include an emphasis on energy, as energy is the single most important human activity associated with greenhouse gas emissions, the tracking of capital stocks by vintage and technology with particular emphasis on power generation (the sector responsible for the largest share of CO_2 emissions, the production of fossil fuel resources using a resource-reserve framework, the calculation of emissions and emissions mitigation for a full suite of 15 greenhouse-related gases, and global coverage of greenhouse gas emissions.

The layout of this documentation is as follows. Section one provides an introduction and brief overview. Section two defines the sets used in the model. This is useful for describing the sets over which different equations are defined, and for summations over the elements of different sets. Section three gives an overview of price relationships in the model.

Section four deals with how production is handled in the model. The first part of this section discusses the profit maximization decision firms' face given a fixed capital stock. This is followed by a description of the investment decision in the model at the sector level. Next, this section covers investment at the subsector level. Subsequently, this section discusses retained earnings, clearing capital markets, and the production of capital goods. Section five describes production dynamics. This incorporates how technical change is handled in the model, and how vintages of capital transition from new vintages with a long-run elasticity of substitution to old vintages with a short-run elasticity of substitution.

Section six discusses the household sector of the model. This section covers household labor supply, household income, household savings, and household final demand. Additionally, this section discusses some of the properties of the demand system. Section seven covers the government sector of the model. This includes a description of government revenue, government savings, and government expenditures. Section eight deals with how trade is modeled. This includes a description of how trade operates when the model is run with a single region, and a description of trade under multi-region operation.

Section nine describes the equilibrium conditions of the model. These include the commodity market clearing conditions, the factor market clearing conditions, and the income balance conditions. Additionally, this section describes the test to verify that Walras' Law is satisfied, and a discussion of the numeraire. Finally, this section covers counting equations and endogenous variables in the model.

Section ten covers greenhouse gas emissions in the model. This includes both CO_2 emissions and non-CO_2 greenhouse gas emissions, as well as a discussion of the carbon price in the model. Finally, at the end of the document are lists of all the endogenous variables, exogenous variables, parameters, and sets used in the model.

2. DEFINITIONS OF SETS

In describing the equations of the model, it will be very useful to define several sets. There are four classes of sets that will prove useful: sets covering the inputs to production; sets covering the outputs of production; sets covering the type of investment closure for markets; and sets covering the types of trade closure for markets.

The following sets cover the types of inputs to production:

1. *Inputs* a set of all inputs to production.
2. *Var* a set of all variable inputs to production.

The set of inputs to production includes all commodities, plus the primary factors of production, labor and capital. The set *Var* is a subset of *Inputs* and the only difference between the two sets is that the fixed input capital is not included among the set of variable inputs.

The following sets cover the types of outputs of production

3. *Sec* a set of all the sectors, or all the types of goods or commodities
4. *SecSub* a set of all sectors and subsectors, or all ways of producing goods
5. *All* a set of all sectors, subsectors, and vintages, or a set of all profit maximizing entities.
6. Sub_j a set of all the subsectors within a particular sector $j \in Sec$.

The set of all sectors covers all the goods or commodities consumed in the model, the set of all sectors and subsectors covers all the different ways of producing those goods, and the set of all sectors, subsectors, and vintages covers all the different profit maximizing entities in the model, which include all the different vintages of capital contained within each sector and subsector. Finally, each sector may contain a different number of subsectors, so the sets Sub_j cover all the subsectors in each sector j. To date, subsectors have only been used in the electricity sector, so for all other sectors, Sub_j contains a single subsector. The union of all of the Sub_j sets is equal to the set *SecSub*.

The following sets cover the different methods for determining investment levels:

7. *OutAcc* a set of all sectors that use the investment accelerator method.
8. *InvAcc* a set of all sectors that use the output accelerator method.

Investment levels in different sectors are determined by one of two accelerator methods. The output accelerator method is used for the energy intensive sectors, and the investment accelerator method is used for all other sectors. The *OutAcc* set and the *InvAcc* set are both subsets of *Sec*, the set of all sectors. Also, the intersection of these two sets is the empty set, and the union of these two sets is *Sec*.

The following sets cover the methods for modeling trade in different sectors:

9. *OpenMkt* a set of all sectors whose goods are traded in an open market.
10. *CloseMkt* a set of all sectors whose goods are traded in a closed market.

Each commodity in the model may either be traded in a closed market with an exogenous level of net exports, or in an open market with an endogenous level of net exports. The *OpenMkt* and the *CloseMkt* sets are both subsets of the set of all commodities, *Sec*, and the intersection of these two sets is the empty set, and their union is again the set of all commodities, *Sec.*

When running the model with multiple regions, there are a few additional sets that need to be defined:

11. *Regions* a set of all regions in the model.
12. *OpenMktF* a set of all open markets with a fixed world price.
13. *OpenMktE* a set of all open markets with an endogenous world price.

When running the model with multiple regions, all of the equations in this documentation described outside of section 8.2, which describes multiple region operation, are defined over all the regions of the model, or all elements of the set *Regions*.

Finally, there are several sets that will be useful in describing emissions calculations in the model.

14. *Sources* a set of all sources of greenhouse gas emissions.
15. $NonCO_2$ a set of all sources of non-CO_2 greenhouse gas emissions.
16. $SecSource_i$ a set of greenhouse gas emissions sources associated with a particular sector $i \in Sec$.

There are multiple sources of greenhouse gas emissions in the model, and all of these are contained in the set, *Sources*. A subset of this set is the set of sources associated with non-CO_2 greenhouse gas emissions. Since non-CO_2 emissions are treated differently from CO_2 emissions, it is useful to define the set $NonCO_2$ as the set of sources of non-

CO_2 greenhouse gas emissions. Finally, for each sector $i \in Sec$ it is helpful to define the set of sources of greenhouse gas emissions that are associated with that sector, $SecSource_i$.

3. PRICE RELATIONSHIPS

There are several price relationships that can be described to relate market prices to the after tax prices paid by demanders and the after tax prices received by suppliers. These price relationships illuminate the tax wedges that come between price paid and price received. While the model code makes explicit use of these price relationships, the rest of this document generally uses the market price instead of the price paid and price received variables.

The first price relationship shows how the after tax price received by suppliers, or producers, is related to the market price:

$$p^{rec}{}_j = \frac{p_j}{\left(1 + tx_{ibt,j}\right)} \tag{1}$$

where $p^{rec}{}_j$ is the price received by the suppliers of product j ; p_j is the market price of product j; and $tx_{ibt,j}$ is the indirect business tax rate for sector j, which has a nominal incidence that falls on the producer, so the quotient of the market price and one plus the indirect business tax rate is the price received by the producer.

The next price relationship is that between the after tax price paid by demanders, or consumers, of a particular good, and the market price:

$$p^{paid}{}_i = p_i + cf_i \tag{2}$$

where $p^{paid}{}_i$ is the price paid by demanders of input i ; p_i is the market price of input i; and cf_i is the additive carbon fee associated with input i.

The final price relationship presented in this section is how the after tax wage rate is related to the market price of labor. The following equation describes this relationship:

$$w = p_{i=labor} \cdot \left(1 - tx_{labor}\right) \tag{3}$$

where w is the after tax wage rate; $p_{i=labor}$ is the price of labor; and tx_{labor} is the proportional tax rate on labor, or social security tax. The after tax wage rate, w, is explicitly used later in this documentation.

The model allows for a wide variety of other tax instruments; however the set described above are the ones most often used, and hence the remainder of this documentation limits itself to this set of tax instruments for the sake of clarity.

4. PRODUCTION

Within each model time step, the firm's decision is broken into two stages: an investment decision, where capital is allocated across production sectors; and a profit maximization problem, where variable inputs are selected to maximize profits subject to the fixed capital stock. During the profit maximization problem, each vintage of the firm's capital stock is fixed. Oldest vintages are retired if their lifetimes have expired, and the firm operates its existing fixed capital stock. The firm's problem is to maximize profits through its choice of variable inputs. Before the profit maximization problem, the firm may invest in new capital, so while the older vintages of capital remain in place, the firm's investment decision is made to choose the capital stock for the newest vintage. That newest vintage of capital will then also be operated in the firm's profit maximization problem. The investment decision allocates capital amongst competing uses, and once capital is in place, it cannot be transformed for use in any other sector or subsector.

When the vintages of capital are operated, there are separate production functions, and thus separate profit maximizations, for each sector, subsector, and vintage of capital. The result of these profit maximization problems is a set of factor demand equations. Each sector, subsector, and vintage of capital has a factor demand equation for every variable input. Furthermore, this stage of the firm's decision results in a single supply equation for each sector, subsector, and vintage of capital. These will all feed into the excess demand equations that will be discussed in a later section.

The firm's investment decision forms the other stage of the firm's decision problem. Each sector and subsector must decide upon the level of investment required to achieve the desired capital stock for the newest vintage.[3] This decision allocates capital across different uses, taking into account the expectations of that capital's profitability over its lifetime in the various different uses. From the firm's investment decision, the factor demand for the fixed factor, capital, emerges.

We also see in this section how capital is produced. This will determine how total investment is allocated to the different rows in the investment column of the final demand portion of the social accounting matrix.

In addition to the demand for capital and the production of capital, this section describes one aspect of the supply of capital. The total amount of investment must be balanced by the total amount of savings by households, the government, and firms, plus external borrowing. This is accomplished by adjustment of the interest rate in the model to ensure that the market clears. Retained earnings, or corporate savings, are discussed in this section.

[3] Since the model operates in five year time steps, current period investment does not directly result in the capital stock of the newest vintage. Current period investment, like other flows, is an annual amount, and thus represents investment in a single year. The current vintage of the capital stock, on the other hand, represents the accumulation of investment over the entire five year period represented by the time step. In order to compute the level of the current vintage of the capital stock, investments in the years between those explicitly represented by the five year time steps are linearly interpolated from current investment and the investment from the previous period.

4.1 Profit Maximization Decision

The current implementation of the production function in the model is a flat, non-nested constant elasticity of substitution (CES) production function of the following form: [4]

$$q_j = \alpha_{0j}\left[\sum_{i \in Inputs}\left(\alpha_{ij}x_{ij}\right)^{\rho}\right]^{\frac{1}{\rho}} \quad \forall j \in All \tag{4}$$

where q_j is the quantity of output for sector j; x_{ij} is sector j's factor demand for input i; the α's are technical coefficients of the production function; and ρ is equal to $(\sigma-1)/\sigma$, where σ is the elasticity of substitution.[5]

Given its fixed inputs, the firm's decision is to maximize profits[6] through choice of variable inputs:

$$\max_{x_{ij}|_{i \in Var}} \pi_j = \frac{p_j}{\left(1+tx_{ibt,j}\right)}\alpha_{0j}\left[\sum_{i \in Inputs}\left(\alpha_{ij}x_{ij}\right)^{\rho}\right]^{\frac{1}{\rho}} - \sum_{i \in Var}\left(p_i + cf_i\right)x_{ij} \quad \forall j \in All \tag{5}$$

where π_j is profits for firm j; p_j is the market price of output j;[7] $tx_{ibt,j}$ is the indirect business tax rate for sector j, which has a nominal incidence that falls on the producer, so the quotient of the market price and one plus the indirect business tax rate is the price received by the producer; p_i is the price of input i; and cf_i is the additive carbon fee associated with input i.[8] Because of the vintaged structure of the capital stock, the amount of capital used in production, x_{Nj}, is not a choice variable in the firm's profit maximization problem. Production for each vintage occurs using the fixed amount of capital available for that vintage. For the current period, production occurs using the

---

[4] The model utilizes separate production functions for each sector, subsector and vintage of the capital stock; however, while the distinction between sector, subsector, and vintage are important for the firm's investment decision, for the firm's profit maximization problem they can all be compressed into a single index. Therefore, subscripts for subsectors and vintages are being suppressed at this point, and the j index, while technically an index over sectors, in this section is used as an index over sectors, subsectors, and vintages. The total number of sector, subsector, and vintage and vintage specific profit maximizing entities is N_{all}.

[5] The subscripts for sector, subsector, and vintage on σ and ρ have been suppressed here. However, it is important to note that within a given sector and subsector, the model utilizes two different values for σ, a long-run elasticity for the current vintage, and a short-run elasticity for the existing vintages. The transformation of the long run elasticity of substitution into the short run elasticity of substitution is discussed in the sector on production dynamics. Also, in some sectors and vintages the elasticity of substitution, σ, is equal to zero and the production function then becomes Leontief.

[6] Note that profits are defined here as revenue minus the cost of variable inputs, thus the cost of the fixed capital stock is not included in the profit calculation. Profit here can be considered short-run profit or restricted profit.

[7] Note that the price of output for all subsectors and vintages within a particular sector is the same. Therefore the number of prices in the model is equal to the number of elements in the set *Inputs*.

[8] The carbon fee associated with input i, cf_i, is essentially the price of the carbon content in one unit of input i. This can be thought of as the emissions allowance price in a carbon cap and trade program, or as a carbon tax.

current vintage of capital. The investment decision that determines the amount of capital in each vintage will be discussed below.

The first order conditions that result from the profit maximization problem are as follows:

$$\frac{\partial \pi_j}{\partial x_{kj}} = \frac{p_j}{\left(1+tx_{ibt,j}\right)} \alpha_{0j} \left[\sum_{i \in Inputs} \left(\alpha_{ij} x_{ij}\right)^{\rho} \right]^{\frac{1-\rho}{\rho}} \alpha_{kj} \left(\alpha_{kj} x_{kj}\right)^{\rho-1} - \left(p_k + cf_k\right) = 0 \qquad \begin{matrix} \forall k \in Var \\ \forall j \in All \end{matrix} \tag{6}$$

where k is an index over inputs, and the other variables are as defined above. From the first order conditions, it is possible to solve for the factor demand equations:

$$x_{ij} = \alpha_0^{\frac{1}{1-\rho}} \alpha_{ij}^{\frac{\rho}{1-\rho}} \left(\frac{p_j / \left(1+tx_{ibt,j}\right)}{p_i + cf_i} \right)^{\frac{1}{1-\rho}} \left(\frac{V_j}{Z_j} \right)^{\frac{1}{\rho}} \qquad \forall i \in Var,\ \forall j \in All \tag{7}$$

where,

$$V_j \equiv \left(\alpha_{Nj} x_{Nj}\right)^{\rho} \qquad \forall j \in All \tag{8}$$

and,

$$Z_j \equiv 1 - \left(\alpha_{0j} \frac{p_j}{\left(1+tx_{ibt,j}\right)} \right)^{\frac{\rho}{1-\rho}} \sum_{i \in Var} \left(\frac{p_i + cf_i}{\alpha_{ij}} \right)^{\frac{\rho}{\rho-1}} \qquad \forall j \in All \tag{9}$$

These factor demand equations will form part of the excess demand equations that will be discussed later in this document. Substituting the factor demand equations back into the profit function and simplifying results in:

$$\pi_j = \alpha_{0j} \frac{p_j}{\left(1+tx_{ibt,j}\right)} V_j^{\frac{1}{\rho}} Z_j^{\frac{\rho-1}{\rho}} \qquad \forall j \in All \tag{10}$$

From the profit function, we can apply Hotelling's lemma[9] to find the firms' outputs:[10]

$$q_j = \left(\frac{V_j}{Z_j} \right)^{\frac{1}{\rho}} \qquad \forall j \in All \tag{11}$$

---

[9] Hotelling's lemma: let $y_i(p)$ be the firm's net supply function for good i. Then $y_i(\mathbf{p}) = \partial \pi(\mathbf{p}) / \partial p_i$ for $i=1,\ldots,n$, assuming that the derivative exists and $p_i>0$.

[10] For older vintages, production is shut down if the profit rate falls to zero, and at low profit rates approaching zero, production is scaled back.

These supply equations will also be used in the excess demand equations.

4.2 Investment at Sector Level

Demand for capital is determined by the investment decision in the model. The supply of capital is determined by the savings decisions of the household and government, which will be discussed later sections of this document, and by the savings decisions of firms in the form of retained earnings, or corporate savings. The market is cleared by adjusting the interest rate, which affects the expected profitability of capital, and the supply of savings.

The investment decision for the firm is to decide upon the level for the fixed factors of production in the new production vintage. More specifically, the firm must set investment to choose the amount of new capital stock. At the sector level, there are two methods for determining investment in the model, one used in the energy transformation sectors,[11] and another used in all other sectors. Both of these methods require some formulation of expectations so that the expected future profit rate is known.

The method used for allocating investment in the model differs from standard practice in that returns to investment are not equalized across investment opportunities in different sectors.[12] The accelerator methods described below move investment towards sectors with higher expected profit rates, but investment is not moved to the point where expected profit rates are equalized across sectors. One of the reasons for using the accelerator methods instead of a zero profits condition is that the model needs to capture the speed at which new technologies can penetrate the market. If expected profit rates are allowed to equalize across sectors, and no capital adjustment costs are included, then profitable infant industries can achieve market penetration in the model too quickly.

4.2.1 Expected Future Profit Rate

The key to calculating the current period investment using both the investment accelerator and the output accelerator method is calculating expected future profits.

$$e\pi_{j,t} = \sum_{yr=1}^{T_j} \left(\frac{\pi_{j,yr}}{1 + r_{j,t}} \right)^{yr} \quad \forall j \in Sec \tag{12}$$

where $e\pi_{j,t}$ is expected future profits for the newest vintage of sector j in period t; yr is an index over years instead of periods; T_j is the capital lifetime in sector j measured in years; $\pi_{j,yr}$ is profit in sector j in year yr that is consistent with expected future prices; and $r_{j,t}$ is the interest rate in sector j during period t. The calculation described in equation (12) is rather cumbersome, for among other things, it requires profits in years not explicitly represented in the model to be approximated. So instead, the model obtains an

[11] The energy transformation sectors include coke, electricity generation, refined oil and petroleum products, and distributed natural gas.

[12] A separate version of the model has been run with a zero profits condition that equalizes the returns to new investment across investment opportunities in different sectors; however, this option has not been made available in the full model.

approximation of this value by substituting a set of present discounted prices into the profit function. This approximation is exact when $\rho/(1-\rho) = 1$.[13]

The expected present discounted prices are calculated by multipling current prices by the following factor:

$$fac_{i,j} = \sum_{yr=1}^{T_j} \left(\frac{1 + pc_{i,yr}}{1 + r_{j,t}} \right)^{yr} \quad \forall i \in Var \,, \ \forall j \in Sec \tag{13}$$

where $fac_{i,j}$ is the factor used to convert current prices to present value of a discrete stream of future prices for product i in sector j;[14] $pc_{i,yr}$ is the percentage change in the price of product i in year yr from the current period price, to date the model has always been run with $pc_{i,yr}$ set equal to zero, which implies myopic expectations;[15] T_j is the lifetime of capital in sector j, measured in years; and $r_{j,t}$ is the sector specific interest rate for sector j in period t. This is calculated as:

$$r_{j,t} = r_t + \omega_j \quad \forall j \in Sec \tag{14}$$

where r_t is the interest rate in period t, and ω_j is the "investment wedge," a sector-specific adder to the market interest rate.[16] Since the interest rate used to calculate the factor in equation (13) is sector specific, and the expected lifetime of capital is sector specific, the factor used for input prices varies depending on the sector using the input.

Expectations about future carbon policy changes can be explicitly represented in the present discounted carbon fee:

$$cfac_{i,j} = \sum_{yr=1}^{T_j} \left(\frac{1 + cfc_{i,yr}}{1 + r_{j,t}} \right)^{yr} \quad \forall i \in Var \,, \ \forall j \in Sec \tag{15}$$

where $cfac_{i,j}$ is the factor for converting the current carbon fee to the present value of a discrete stream of future carbon fees in sector j associated with input i; T_j is the lifetime of capital in sector j, measured in years; $r_{j,t}$ is the sector specific interest rate for sector j in period t; and $cfc_{i,yr}$ is the percentage change in the carbon fee for input i in year yr from the current period carbon fee. Note that while the carbon fee for each input, cf_i, is not sector specific, $cfac_{i,j}$ is sector specific. This is because each sector potentially has a different lifetime of capital, T_j, and a different sector specific interest rate, $r_{j,t}$.

Utilizing the factors in the profit function from equation (10) yields:[17]

---

[13] This corresponds to an elasticity of substitution $\sigma = 2$.

[14] The factor for the output of sector j can be represented as $fac_{j,j}$.

[15] The expected price change parameter for the fixed input capital, $pc_{i=N,yr}$ is equal to zero for all years.

[16] $\omega_{j\,j}$ is used in the model calibration process, and is discussed in (Sands and Fawcett 2005). In short, $\omega_{j\,j}$ is adjusted so that expected profits are equal to one for all sectors in the model base year. In general ω_j does not vary across time periods; however, in some special cases ω_j is increased in later periods to phase out a particular technology.

[17] For scenarios where the indirect business tax is expected to change over time, tx_{ibt} can be worked into the expected price calculation to give an after tax expected price.

$$e\pi_{j,t} = \alpha_{0j} \cdot \frac{p_{j,t}}{\left(1 + tx_{ibt,j}\right)} \cdot fac_{j,j} \cdot V_{j,t}^{\frac{1}{\rho}} \cdot Ze_{j,t}^{\frac{\rho-1}{\rho}} \quad \forall j \in SecSub \tag{16}$$

where $e\pi_{j,t}$ is the expected profit rate for the newest vintage of capital in sector and subsector j; $V_{j,t}$ is the same as defined in equation (8); $tx_{ibt,j}$ is the indirect business tax rate for sector j; ρ is again equal to $(\sigma-1)/\sigma$, where σ is the elasticity of substitution. $Ze_{j,t}$ is simply the Z_j from equation (9) with present discounted prices instead of current period prices:

$$Ze_{j,t} = 1 - \left(\alpha_{0j} \frac{p_j \cdot fac_{j,j}}{\left(1 + tx_{ibt,j}\right)}\right)^{\frac{\rho}{1-\rho}} \sum_{i \in Var} \left(\frac{p_i \cdot fac_{i,j} + cf_i \cdot cfac_{i,j}}{\alpha_{ij}}\right)^{\frac{\rho}{\rho-1}} \quad \forall j \in SecSub \tag{17}$$

If myopic expectations are assumed (i.e. the expected price change parameters, $pc_{i,t}$ and $cfc_{i,t}$, are equal to zero for all inputs, i, and years, t), then the simplified calculation of expected profits in equation (16) is an exact approximation of expected profits in equation (12).[18] The model is presently run exclusively with myopic expectations.

Expected future profits are transformed into the expected future profit rate by dividing expected profits by the value of the factor demand for the fixed factor of production (i.e. capital). The expected future profit rate in period t for sector j, $e\pi rate_{j,t}$, is given by the following equation:

$$e\pi rate_{j,t} = \frac{e\pi Sec}{x_{N,j,v=new} \cdot p_{I,j}} \quad \forall j \in Sec \tag{18}$$

where $x_{N,j,v=new}$ is demand for the newest vintage of capital in sector j; and $p_{I,j}$, which is described below in equation (41), is the price purchase price of capital goods in sector j; and $e\pi Sec_{j,t}$ is sector level expected profits as opposed to subsector specific expected profits. For sectors that only contain one subsector, $e\pi Sec_{j,t}$ is simply equal to $e\pi_{j,t}$, and for sectors that contain multiple subsectors, $e\pi Sec_{j,t}$ is described below in equation (31).

The expected future profit rate can be interpreted as the discounted return on a one dollar investment. An expected profit rate of one implies that investment will break even, covering both operating and capital costs. This is equivalent to setting price equal to unit cost, so an expected profit rate of one satisfies the zero profits condition.[19] A

---

[18] Under myopic expectations $fac_{i,j}$ and $cfac_{i,j}$ are equal to a single constant, fac_j, for values of i. This constant can then be factored out of the expected profits equation, so that expected profits equation can simply be written as, $e\pi_{j,t} = fac_j \cdot \pi_{j,t}$, which is equivalent to the expression for profits in equation (12).

[19] The following equation shows the expected profit rate set equal to one with the assumption of myopic expectations and no taxes made for simplicity:

profit rate of zero means that only operating costs, and no capital costs, are covered. If the expected profit rate falls below zero, then the model will force that vintage of capital to cease operation.

The expected profit rate is calculated every iteration of the model as the interest rate adjusts so that the market for capital clears.[20] This will be discussed further in the household and government sections of this document, which will deal with the supply of capital, and in the section on equilibrium conditions.

4.2.2 Investment Accelerator Method

For all sectors not related to energy transformation, an investment accelerator is used to determine investment levels.[21] Current period investment in this case is based on previous period investment, a base rate of investment growth, the growth in the working age population, and the expected profit rate:

$$I_{j,t} = I_{j,t-1} \cdot base\_rate \cdot \frac{Pop_{WorkAge,t}}{Pop_{WorkAge,t-1}} \cdot e\pi rate_{j,t} \quad \forall j \in InvAcc \tag{19}$$

where $I_{j,t}$ is investment by sector j in period t; $e\pi rate_{j,t}$ is the expected profit rate in sector j in period t; $Pop_{WorkAge,t}$ is the working age population in period t, and $base\_rate$ is used to represent capital deepening, or an overall increase in the amount of capital per worker.

$$e\pi rate_{j,t} = \frac{fac_{N,j} \cdot \pi_{j,t}}{p_{I,j} x_{N,j}} = \frac{fac_{N,j}\left[p_j q_j - \sum_{i \in Var} p_i x_i\right]}{p_{I,j} x_{N,j}} = 1$$

where $fac_{N,j}$ is defined later in equation (25), and is equivalent to the factor from equation (13) under the assumption of myopic expectations; $\pi_{j,t}$ is profit in sector j; $p_{I,j}$ is the purchase price of new capital for sector j, and is specified below in equation (41); and $x_{N,j}$ is the amount of new capital used in sector j. Rearranging yields the equation below:

$$p_j = \frac{\sum_{i \in Var} p_i x_i + \left(\frac{p_{I,j}}{fac_{N,j}}\right) x_{N,j}}{q_j}$$

where the quotient of $p_{I,j}$ and $fac_{N,j}$ is the rental rate of capital. This equation shows that price is equal to unit cost, so if the expected profit rate is set equal to one, then the zero profits condition holds.

[20] Note that except in the base year, the model does not enforce the zero profits condition, so the market for capital clears without the zero profits condition holding. A separate version of the model has experimented with using the zero profits condition instead of the accelerators for investment.

[21] For energy production sectors (i.e. oil production, natural gas production, and coal production) there is an additional wrinkle in the model. Depletable energy resources and reserves are tracked. As long as the resources and reserves are not exhausted, then investment proceeds as described in this section. However, if the planned investment exceeds the available resource or reserves, then investment is scaled back so that the resource or reserve is just exhausted. The model has the capability to track multiple grades of each resource as separate subsectors, and it is possible for the model to track growth of the resource base. In general, the model has been run with a single grade of each resource and the amount of each resource set high enough so that depletion is not an issue. For a detailed description of how the model handles depletable reserves and resources, see Brenkert *et al.* 2005.

With this formulation, investment moves toward sectors with higher expected profit rates. Summing investment across production sectors gives the demand for savings, and the interest rate adjusts each period (and in turn affects the expected profit rate) so that supply and demand for savings are equal.

To find the amount of the fixed factor of production that is used in the profit maximization problem, we need to go from the level of investment to the actual capital stock for the newest vintage. Given the five year time steps currently used in the model, the equation is formulated as follows:

$$x_{N,j,v=new} = 2 \cdot I_{j,t-1} + 3 \cdot I_{j,t} \quad \forall j \in InvAcc \tag{20}$$

where $x_{N,j,v=new}$ is the amount of the fixed input, capital, demanded by the newest vintage of production, *v=new*, of sector *j*.[22]

4.2.3 Output Accelerator Method

Investment in the energy transformation sectors is handled somewhat differently than investment in other sectors. Instead of using the investment accelerator method, investment in the energy transformation sectors is determined using the output accelerator method. The output accelerator method is used instead of the investment accelerator method in these sectors because it is potentially more stable. Investment over time can fluctuate more than output, especially if past patterns of investment are not smooth, or in response to a price shock. With the output accelerator, we first form an estimate of output and then back out the amount of new capital needed to match that level of production. This indirect estimate for investment then adjusts as a function of the expected profit rate.

The first step in using the output accelerator method is to calculate an estimate of gross output for each of the energy transformation sectors:

$$\tilde{q}_{j,t} = q_{j,t-1} \cdot sclinv \quad \forall j \in OutAcc \tag{21}$$

where $\tilde{q}_{j,t}$ is estimated gross output from sector *j* in period *t*; $q_{j,t-1}$ is actual gross output in period *t-1*; *sclinv* is an exogenous scalar multiplier.

Next, the output that is required from new capital is estimated by subtracting the output from existing vintages from an estimate of gross output for the sector:

$$\tilde{q}_{j,t,v=new} = \tilde{q}_{j,t} - \tilde{q}_{j,t,v=old} \quad \forall j \in OutAcc \tag{22}$$

[22] Note that in actual model operation demand for new capital is calculated at the subsector level instead of the sector level, so equation (35) is used instead of equation (20). For sectors that contain a single subsector, the two equations are equivalent. When the sector level demand for new capital is used in calculations for sectors with multiple subsectors (e.g. the sector level expected profit rate calculation from equation (18)), the actual sector level demand for new capital is simply the sum of all subsector demands for new capital.

where $\tilde{q}_{j,t,v=new}$ is the estimated output from the new vintage of capital in sector *j* in period *t*, and $\tilde{q}_{j,t,v=old}$ is the estimated output from old existing vintages of capital in sector *j* in period *t*.

Then, an estimated demand for new capital is simply calculated using the factor demand equation from the CES production function with variable inputs:[23]

$$\tilde{x}_{N,j,v=new} = \alpha_{0j}^{\frac{\rho}{1-\rho}} \alpha_{Nj}^{\frac{\rho}{1-\rho}} \left(\frac{p_j / \left(1 + tx_{ibt,j}\right)}{p_{N,j}} \right)^{\frac{1}{1-\rho}} \tilde{q}_{j,t,v=new} \quad \forall j \in OutAcc \tag{23}$$

where $\tilde{x}_{N,j,v=new}$ is the estimated amount of capital (input *N*) demanded by the newest vintage of production, *v=new*, of sector *j*; p_j is the price of output *j*; $tx_{ibt,j}$ is the indirect business tax rate for sector *j*; and $p_{N,j}$ is the rental price of new capital for sector *j*, which is related to the purchase price of new capital through the following relationship:

$$p_{N,j} = \frac{p_{I,j}}{fac_{N,j}} \quad \forall j \in Sec \tag{24}$$

where $p_{I,j}$ is the purchase price of new capital for sector *j*, as defined below in equation (41); and $fac_{N,j}$ is defined as follows:

$$fac_{N,j} = \sum_{yr=1}^{T_j} \left(\frac{1}{1 + r_{j,t}} \right)^{yr} \quad \forall j \in Sec \tag{25}$$

where T_j is the lifetime in years of capital in sector *j*; and $r_{j,t}$ is the interest rate in sector *j* during period *t*, as given by equation (14).

Next, estimated investment can be backed out from the estimated amount of capital in the new vintage and the previous vintage:[24]

$$\tilde{I}_{j,t} = \frac{\tilde{x}_{N,j,v=new} - 2 \cdot I_{j,t-1}}{3} \quad \forall j \in OutAcc \tag{26}$$

In order for investment to respond to the expected profit rate, actual investment is calculated set equal to the product of estimated investment and the expected profit rate.[25]

[23] For sectors that contain multiple subsectors (i.e. electricity generation in the current version of the model) determining the amount of new capital is more involved. Exactly how this is done will be discussed in the treatment of subsectors in section *4.3.4*, which covers subsector level investment.

[24] If the estimated investment calculated in equation (26) is smaller than ten percent of the estimated demand for capital from equation (23), then instead of using equation (26) to set estimated investment, it is simply set equal to ten percent of the estimated demand for capital from equation (23).

$$I_{j,t} = \tilde{I}_{j,t} \cdot e\pi rate_{j,t} \quad \forall j \in OutAcc \tag{27}$$

This adjustment allows investment to move towards sectors with high expected profit rates, and away from sectors with low expected profit rates. Finally, equation (20) can be used to calculate the actual capital stock for the newest vintage.

Finally, we calculate the actual capital stock for the newest vintage. Given the five year time steps currently used in the model, the equation is formulated as follows:

$$x_{N,j,v=new} = 2 \cdot I_{j,t-1} + 3 \cdot I_{j,t} \quad \forall j \in OutAcc \tag{28}$$

where $x_{N,j,v=new}$ is the amount of the fixed input, capital, demanded by the newest vintage of production, $v=new$, of sector j. [26]

4.3 Subsector Investment Shares

Much of the technology detail in the model is captured through the use of subsectors. All of the subsectors within an individual sector produce a homogeneous product; however they can use different production technologies to produce that product. Perfect competition amongst profit maximizing subsectors would generally result in one technology capturing the entire market share. However, in the real world we often observe multiple technologies being used simultaneously within a sector (e.g. coal, natural gas, oil, nuclear, hydro, and a host of renewable technologies all used for the production of electricity). If we further disaggregate the product we may see that these different technologies are actually serving different portions of the market (e.g. peak versus base load electricity, or power generation in different regions). However, in order to represent all of the technology detail at the level of product aggregation in the model, we need to treat the competition among subsectors somewhat differently. The way this is accomplished in the model is the use of a logit sharing mechanism for allocating investment among subsectors.[27]

The previous section discussed two methods for determining sector level investment. The model also must determine the level of investment occurring at the subsector level. There are three aspects of subsector level investment that will be discussed here: first, how sector level expected profits are determined in the presence of subsectors; second, how total sector level investment is shared out among subsectors; and

[25] Sector level expected profit is determined in a slightly different way in sectors that have more than one subsector (i.e. the electricity sector in the current version of the model). This will be discussed below in section *4.3.2* on sector level expected profits.

[26] Note that in actual model operation demand for new capital is calculated at the subsector level instead of the sector level, so equation (35) is used instead of equation (28). For sectors that contain a single subsector, the two equations are equivalent. When the sector level demand for new capital is used in calculations for sectors with multiple subsectors (e.g. the sector level expected profit rate calculation from equation (18)), the actual sector level demand for new capital is simply the sum of all subsector demands for new capital.

[27] Clarke and Edmonds (1992) gives an overview of the use of a logit sharing mechanism for modeling energy technologies.

third, how demand for new capital is determined using the output accelerator method in sectors that contain multiple subsectors.

4.3.1 Logit Share Calculation

All three of these topics hinge on how shares are determined at the subsector level using the logit sharing mechanism:[28]

$$share_{jj} = \frac{b_{jj} C_{jj}^{\lambda}}{\sum_{k} b_k C_k^{\lambda}} \quad \forall jj \in Sub_j \,,\ \forall j \in Sec \tag{29}$$

where $share_{jj}$ is the share for subsector jj, C_{jj} is the variable the shares are based upon, b_{jj} is a calibration parameter for subsector jj, λ is a parameter that determines the rate one technology can substitute for another, and both k and jj are used here as indexes over all subsectors.[29] The variable C_{jj} may be either expected profits in subsector jj, as described in equation (16), or levelized cost in subsector jj.[30] When levelized cost is used, C_{jj} is calculated as follows:

$$C_{jj} = \frac{1}{\alpha_{0,jj}} \left[\sum_{i \in Inputs} \left(\frac{p_i + cf_i}{\alpha_{i,jj}} \right)^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}} \quad \forall jj \in Sub_j \,,\ \forall j \in Sec \tag{30}$$

and C_{jj} is simply the unit cost function for a CES production function with all variable inputs;[31] the α*'s* are parameters of the CES production function;[32] p_i is the price of input i;[33] cf_i is the carbon fee associated with input i; and ρ is again equal to $(\sigma-1)/\sigma$, where σ is the elasticity of substitution.

4.3.2 Sector Level Expected Profits

The first use of the logit shares is to calculate sector level expected profits in sectors that contain more than one subsector. Once the shares are calculated for all of the subsectors, sector level expected profit can be found. Equation (16) can be used to find

---

[28] The logit share mechanism is used for subsectors with endogenous investment. The shares for subsectors with exogenous investment are equal to zero.

[29] The subsectors may also be nested, in which case equation (29) is used at each node of the nest, and k is instead an index over the leaves and nodes below the node for which the share is being calculated.

[30] Currently, the United States region uses levelized cost, while other regions in the model use expected profits in the logit sharing mechanism.

[31] In the electricity sector, it is sometimes useful to convert the units used for levelized cost to mills per kWh. This can be accomplished by simply dividing by *elecfac*, which is a conversion factor from units of output in real dollars to units of output in terawatt hours (TWh). Levelized cost, C_{jj}, divided by *elecfac* is thus measured in mills per kWh.

[32] Note that since we are explicitly dealing with subsectors in this section, the notation for the α parameters looks slightly different here. Instead of the subscript j representing the index over all sectors, subsectors, and vintages; here, we are using the subscript jj to emphasize that we are looking at a specific subsector. The full set of subscripts would be $\alpha_{i,j,jj,v,t}$ but here we are suppressing all but the i and jj subscripts.

[33] The price of input N is the rental price of capital as given in equation (24).

expected profits for each individual subsector. Then sector level expected profit can be found simply summing the product of subsector level expected profit and the subsector share over all subsectors.

$$e\pi Sec_{j,t} = \sum_{jj \in Sub_j} share_{jj} \cdot e\pi_{jj,t} \quad \forall j \in Sec \tag{31}$$

where $e\pi Sec_{j,t}$ is expected profits for the newest vintage of capital in sector j in period t; and $e\pi_{jj,t}$ is expected profits for the newest vintage of capital in subsector jj in period t, and is calculated as shown in equation (16). This calculation yields an exact measure of expected profits when the C_{jj} variable that is used in the logit sharing mechanism, equation (29), is defined as expected profits. When levelized costs are used in the logit sharing mechanism, equation (31) is an approximation of sector level expected profits.

4.3.3 Sector Level Demand for New Capital

The logit shares are also used in finding the amount of new capital required when using the output accelerator method in sectors with multiple subsectors. For each subsector, we can calculate the unit factor demand, or capital output ratio:

$$a_{N,jj} = \alpha_{0,jj}^{\frac{\rho}{1-\rho}} \alpha_{N,jj}^{\frac{\rho}{1-\rho}} \left(\frac{p_{jj} / \left(1 + tx_{ibt,jj}\right)}{p_{N,j}} \right)^{\frac{1}{1-\rho}} \quad \forall jj \in Sub_j\,,\ \forall j \in Sec \tag{32}$$

where $a_{N,jj}$ is the capital output ratio for subsector jj; $p_{N,j}$ is the price of input N in sector j, which is the rental rate of capital in sector j, of which jj is a subsector; p_{jj} is the price of output from subsector jj; and $tx_{ibt,j}$ is the indirect business tax rate for subsector jj.

Since the product of the sector level demand for new capital and the subsector share is the subsector level demand for new capital, this product may be divided by the subsector level capital output ratio found in equation (32) to obtain the subsector output. The result may then be summed over subsectors to obtain sector level output. By rearranging this calculation, the subsector capital output ratios can be used in combination with the subsector shares and the estimated output from the new vintage of capital to obtain the estimated sector level demand for new capital:

$$\tilde{x}_{N,j,v=new} = \left(\sum_{jj \in Sub_j} \frac{share_{jj}}{a_{N,jj}} \right)^{-1} \cdot \tilde{q}_{j,t,v=new} \quad \forall j \in OutAcc \tag{33}$$

From here, the procedure for calculating actual investment and demand for new capital is the same as described in the section above on the output accelerator method.

4.3.4 Subsector Level Investment

The final use of the logit shares is to share sector level investment among subsectors. After the sector level investment is calculated, the subsector level investment is simply the product of the subsector share and the sector level investment.

$$I_{jj,t} = I_{j,t} \cdot share_{jj} \quad \forall jj \in Sub_j ,\ \forall j \in Sec \tag{34}$$

where $I_{jj,t}$ is subsector level investment in period t, and $I_{j,t}$ is sector level investment in period t.

Finally, we calculate the actual capital stock for the newest vintage. Given the five year time steps currently used in the model, the equation is formulated as follows:

$$x_{N,jj,v=new} = 2 \cdot I_{jj,t-1} + 3 \cdot I_{jj,t} \quad \forall jj \in Sub_j ,\ \forall j \in Sec \tag{35}$$

where $x_{N,jj,v=new}$ is the amount of the fixed input, capital, demanded by the newest vintage of production, $v=new$, of subsector jj.

4.4 Retained Earnings

Profits earned by firms are put to three uses: the payment of dividends, which are part of household income; the payment of taxes, which are part of government revenue; and corporate savings, here referred to as retained earnings, which along with personal savings, government savings, and the deficit in the balance of payments on current account form the supply of capital. The corporate savings rate, or retained earnings rate is calculated as follows:

$$s_{\pi,j} = remax_j \cdot \left(1 - \delta re_j \cdot e^{\phi re_j \cdot r}\right) \quad \forall j \in All \tag{36}$$

where $s_{\pi,j}$ is the rate of corporate savings or retained corporate earnings in sector, subsector, and vintage j; $remax_j$ is a parameter representing the maximum rate of retained earnings in sector, subsector, and vintage j; ϕre_j is a sector specific parameter that determines the sensitivity of corporate savings to the interest rate r; and δre_j is a scale parameter. Total corporate savings is simply the product of profits (π_j) and the rate of corporate savings ($s_{\pi,j}$), summed over all production entities:

$$S_{RE} = \sum_{j \in All} \pi_j \cdot s_{\pi,j} \tag{37}$$

where S_{RE} is total corporate savings, or retained earnings.

4.5 Clearing Capital Markets

In order for the capital market to clear, the supply of savings must be equal to the demand for savings, also known as demand for investment. The market interest rate adjusts to bring this market into equilibrium. As the interest rate increases, the supply of savings increases, as described in equation (36) in regards to corporate savings, and in later sections in reference to household and government savings. Furthermore, investment decreases as the interest rate increases. This happens because, as the interest rate rises, expected profits fall (as can best be seen in equation (12)), and as expected profit rates fall, new investment falls (see equations (19) and (27)). The interest rate adjusts during each iteration of the model until the market clears.

One point of departure from most CGE models is the use of the zero profits condition. As noted earlier, the condition in the model that the expected profit rate be set equal to one is equivalent to the standard zero profits condition. In the base year, the sector specific interest rates are calibrated so that the expected profit rate is one in all sectors, and thus the zero profits condition holds. However, in subsequent periods, this condition is not enforced. The market interest rate adjusts to clear the capital market, but there is no mechanism in the model that enforces the zero profit condition during model operation.

4.6 Production of Capital Goods

One additional aspect of investment in the model involves determining what inputs are used in the production of capital goods. In other words, how are the investments made by various firms distributed between the different rows of the investment column in the social accounting matrix. In order to describe how this is done in the model, first we must describe a new variable, $Inv_{i,j,t}$, which is investment by sector j spent on input i in period t, where the following relationship holds:[34]

$$I_{j,t} = \sum_{i \in Sec} Inv_{i,j,t} \quad \forall j \in Sec \tag{38}$$

Similarly, if we sum $Inv_{i,j,t}$ over all j sectors, the result is the entry for the i^{th} row of the investment column in the SAM.

Production of the capital good, $I_{j,t}$, occurs through a Leontief production function:

$$I_{j,t} = \min_{Inv_{i,j,t}|_{i \in Sec}} \left\{ \frac{Inv_{i,j,t}}{CapMat_{ij}} \right\} \quad \forall j \in Sec \tag{39}$$

where $CapMat$ is an exogenously determined matrix of Leontief coefficients. So, if $I_{j,t}$, investment by sector j in period t, is known from the investment calculations above, then $Inv_{i,j,t}$ can be calculated as:

$$Inv_{i,j,t} = I_{j,t} CapMat_{ij} \quad \forall i \in Sec, \ \forall j \in Sec \tag{40}$$

where $CapMat_{ij}$ can be interpreted as the amount of input i required to produce one unit of capital in sector j.

We can also now define the price index for investment, $p_{I,j}$:

$$p_{I,j} = \sum_{i \in Sec} CapMat_{ij} \cdot \left(p_i + cf_i \right) \quad \forall j \in Sec \tag{41}$$

---

[34] Note that the shares calculated using equation (29) can be used to share sector level $Inv_{i,j,t}$ to the subsector level.

where p_i is the price of good i; and cf_i is the carbon fee associated with good i. This price index for investment, $p_{I,j}$, is the purchase price of capital in sector j.

5. PRODUCTION DYNAMICS

This section covers how the coefficients of the production function change over time. One aspect of this is how technical change is represented in the model. Technical change takes the form of both neutral technical change, and potentially, a non-neutral factor augmenting technical change.

The other aspect of production dynamics discussed in this section, is how the elasticities of substitution change as new vintages of capital become old vintages of capital. New vintages of capital operate as fully variable production technologies with a long-run elasticity of substitution, while old vintages of capital operate as quasi-fixed production technologies with a short-run elasticity of substitution. This means that new vintages are more elastic than old vintages. Since the elasticity of substitution changes as a vintage transforms from new to old, the α coefficients of the production function must change in order to maintain the same factor proportions at current prices.

5.1 Technical Change

In most top-down models, technical change is achieved through changes of the parameters of the production function. Models that have a bottom-up orientation have a more explicit representation of different technologies, and thus may model technical change from an engineering perspective of explicit changes to a particular technology, or the introduction of entirely new technologies. The SGM incorporates aspects of both types of models, and thus utilizes both types of technical change.

There are two ways technology (i.e. the relationship between inputs and outputs) can change in the model. The first is the introduction of a new technology, and/or the removal of an old one. One example of this type of change is the introduction of coal integrated gasification combined cycle electricity generation (coal IGCC). Before a certain period, the coal IGCC subsector of the electricity generation sector is simply turned off.

The second type of technical change is the incremental enhancements to the contribution inputs make to the production process. This type of technical change is represented by the shifting of an isoquant. There are two forms this type of technical change may take, neutral or non-neutral.

Neutral technical change occurs when the efficiency of all inputs to a production process change at the same rate. The result is that if relative prices remain the same, the input mix will not change. Changes in the α_{0j} parameters of the CES production functions result in neutral technical change. This is implemented in the model through the following equation[35]:

[35] Note that the t subscripts on the α parameters in the above sections were suppressed.

$$\alpha_{0,j,t} = \alpha_{0,j,t=0} \prod_{s=1}^{t} \left(1+\gamma_{0,j,s}\right)^{NStep} \quad \forall j \in SecSub \tag{42}$$

where t represents the model time period, and $t=0$ is the initial period. *NStep* is the number of years in a period, and $\gamma_{0,j,s}$ is an exogenous parameter that represents the rate of neutral technical change for sector j in period s.

Over a long period of time, neutral technical change can result in rates of energy conversion that violate physical laws. To avoid such violations, the model uses neutral technical change sparingly, and relies more heavily on non-neutral technical change. Non-neutral technical change, or factor augmenting technical change, occurs when the efficiency of inputs to a production process each changes at a different rate. In this case, even if relative prices remain unchanged, the input mix will be altered. Non-neutral technical change involves changes to the α_{ij} parameters of the CES production functions:

$$\alpha_{i,j,t} = \alpha_{i,j,t=0} \prod_{s=1}^{t} \left(1+\gamma_{i,j,s}\right)^{NStep} \quad \forall i \in Inputs,\ \forall j \in SecSub \tag{43}$$

where the $\gamma_{i,j,s}$ is an exogenous parameter that represents the rate of technical change for the use of input i in sector j during period s.

5.2 Vintages and the Elasticity of Substitution

As a vintage transforms from new to old, it changes from a long-run fully variable production technology with a relatively elastic long-run elasticity of substitution, to a short-run quasi fixed production technology with a relatively inelastic short-run elasticity of substitution. This transformation occurs between periods of model operation. If the elasticity of substitution changes with no other changes to the production function, then the factor proportions for that vintage will change, even in the absence of price changes or technological change. In order for the factor proportions to remain the same at current prices after the transformation of a vintage from new to old, the α parameters of the production function must be adjusted. The following two equations show how the α*'s* are adjusted to maintain the input-ratios at current prices:[36]

---

[36] In order to derive the equations for the αold's, we start with the new and old vintage unit factor demand equations for a CES production function with variable inputs, and impose that they are both equal to a_{ij}:

$$a_{ij} = \alpha new_{0j}^{\frac{\rho}{1-\rho}} \cdot \alpha new_{ij}^{\frac{\rho}{1-\rho}} \cdot \left[\frac{p_j}{p_i}\right]^{\frac{1}{1-\rho}} \qquad a_{ij} = \alpha old_{0j}^{\frac{\rho^*}{1-\rho^*}} \cdot \alpha old_{ij}^{\frac{\rho^*}{1-\rho^*}} \cdot \left[\frac{p_j}{p_i}\right]^{\frac{1}{1-\rho^*}}$$

where a_{ij} is the unit factor demand, or the amount of input i required to produce one unit of output j; and all taxes have been suppressed. From the second equation above, we solve for αold_{ij}, and substitute the left hand side of the first equation above in for a_{ij}. Simplifying the resulting expression yields:

$$\alpha old_{ij} = \alpha new_{0j}^{\frac{\rho\cdot(1-\rho^*)}{\rho^*\cdot(1-\rho)}} \cdot \alpha new_{ij}^{\frac{\rho\cdot(1-\rho^*)}{\rho^*\cdot(1-\rho)}} \cdot \left[\frac{p_j}{p_i}\right]^{\frac{\rho-\rho^*}{(1-\rho)\rho^*}} \cdot \alpha old_{0j}^{-1}$$

$$\alpha old_{ij} = \alpha new_{ij}^{\frac{\rho}{\rho^*}\frac{(1-\rho^*)}{(1-\rho)}} \left[\frac{p_i}{p_j}\right]^{\frac{\rho-\rho^*}{(1-\rho)\rho^*}} \qquad \forall i \in Var\,,\ \forall j \in All \tag{44}$$

where αold_{ij} is the production function coefficient for old vintages; αnew_{ij} is the production function coefficient for new vintages; p_i is the price of input i; p_j is the price of output j; ρ is equal to $(\sigma-1)/\sigma$, where σ is the long-run elasticity of substitution used for new vintages; and ρ^* is equal to $(\sigma^*-1)/\sigma^*$, where σ^* is the short-run elasticity of substitution used for old vintages. The α_0 parameters of the production function are calculated as follows:

$$\alpha old_{0j} = \alpha new_{0j}^{\frac{\rho}{\rho^*}\frac{(1-\rho^*)}{(1-\rho)}} \qquad \forall i \in Var\,,\ \forall j \in All \tag{45}$$

In other sections of this document, the notation specifying the α coefficients as 'old' or 'new' is suppressed.

6. HOUSEHOLDS

The household sector of the model consists of a single representative household that makes decisions about its supply of labor, its savings, and its demand for final products. Unlike most CGE models, the household sector in SGM does not have an explicit utility theoretic basis. Instead, the model uses the set of household supply and demand equations described in this section.

6.1 Household Labor Supply

Both the wage rate and labor supply are endogenous in the model; however, the household labor leisure decision is not explicitly modeled. Labor supply (q_{labor}) is calculated as a function of the endogenous average annual wage rate (w), the exogenous working age population ($Pop_{WorkAge}$), and exogenous parameters representing the maximum potential share of working age population employed in any given year (θ_{labor}), a scale parameter (δ_{labor}), and a labor supply responsiveness coefficient (ϕ_{labor}).

$$q_{labor} = \theta_{labor} \cdot Pop_{WorkAge} \cdot \left(1 - \delta_{labor} \cdot e^{\phi_{labor} \cdot (w/p_{num})}\right) \tag{46}$$

where the wage rate (w) is divided by the numeraire price (p_{num}) to maintain homogeneity of degree zero in prices. The wage rate is related to the price of labor through the price relationship from equation (3), which is restated below:

$$w = p_{i=labor} \cdot \left(1 - tx_{labor}\right) \tag{47}$$

---

if αold_0 is defined as in equation (45), then the result is the equation for αold_{ij} from equation (44), and the factor proportions for new and old vintages are the same at current prices.

where $p_{i=labor}$ is the price of labor, tx_{labor} is the proportional tax rate on labor.[37] The price of labor, and thus the wage rate, adjusts to ensure that the labor market clears in every time period.

6.2 Household Income

Household disposable income is used in the calculation of savings supply, and in the calculation of household demand for final goods. Households receive income from labor, dividend payments, and government transfers, and households pay taxes to the government. The following equation describes how household disposable income (Y_{dis}) is determined:

$$Y_{dis} = \left[w \cdot q_{labor} + \sum_{j \in All} \pi_j \cdot (1 - tx_\pi) \cdot (1 - s_{\pi,j}) \right] (1 - tx_Y) + TR_{gov} + TR_{carb} \tag{48}$$

where w is the after tax wage rate; q_{labor} is household labor supply; π_j is profit in sector, subsector, and vintage j; tx_π is the profit tax rate; $s_{\pi,j}$ is the rate of corporate savings or retained corporate earnings; tx_Y is the personal income tax rate; TR_{gov} are government transfers; and TR_{carb} are transfers associated with a carbon policy.[38]

Since the household budget constraint is enforced in the calculation of household demand, as described below in section *6.4.1* on the demand function, total household expenditures must be equal to household income. This implies that equation (48), which sets household income equal to the return from factors plus transfer payments, ensures that the household income balance condition is satisfied.

6.3 Household Savings

Household savings is modeled in a similar way to household labor supply. Savings depends upon disposable personal income, the interest rate, and several exogenous parameters. This calculation is slightly more complicated than the labor supply calculation, because unlike the working age population, disposable income is calculated endogenously as described above.

Once the household disposable income (Y_{dis}) is known, household savings can be calculated.

$$S_{hh} = \theta_{hhsave} \cdot Y_{dis} \cdot \left(1 - \delta_{hhsave} e^{\phi_{hhsave} \cdot r}\right) \tag{49}$$

where S_{hh} is household savings supply, θ_{hhsave} is the maximum potential savings rate, δ_{hhsave} is a scale parameter, and ϕ_{hhsave} determines the sensitivity of households to the interest rate r.

[37] This is sometimes referred to as a social security tax, although it does not have all the features of the actual social security tax, such as a maximum amount of income that it may apply to.
[38] TR_{carb} can be calculated in different ways depending how revenue recycling is handled.

6.4 Household Final Demand

While the model does not maximize a utility function to determine household behavior, the household final demand system can be shown to be consistent with utility maximization under certain conditions. The following subsections detail the household final demand specification, and describe some of its properties.

6.4.1 Demand Function

In order to calculate household demand for final goods, the model must know the total value of consumption, which is simply total household income less household expenditures on savings and labor:

$$Y_c = Y_{dis} - S_{hh} - p_{i=labor} \cdot x_{i=labor,hh} \tag{50}$$

where Y_c is the total value of household consumption; Y_{dis} is household disposable income; S_{hh} is household savings supply; $p_{i=labor}$ is the price of labor; and the amount of labor demanded by households, $x_{i=labor,hh}$, is calculated as follows:

$$x_{i=labor,hh} = \eta_{labor,hh} \cdot q_{labor} \tag{51}$$

where q_{labor} is household labor supply as calculated in equation (46), and $\eta_{labor,hh}$ is the household labor demand intensity factor.[39]

The household demand for any final good i can now be calculated via the following equation, using the price elasticity, income elasticity, and an income expenditure normalization:

$$x_{i,hh} = \delta_{i,hh}\left(\frac{p_i + cf_i}{p_{num}}\right)^{\beta_{i,hh}}\left(\frac{Y_c}{p_{num}}\right)^{\gamma_{i,hh}}\left(\frac{1}{\Gamma}\right) \quad \forall i \in Sec \tag{52}$$

where $x_{i,hh}$ is household demand for good i; $\delta_{i,hh}$ is the household demand intensity factor for good i; p_i is the price of good i; cf_i is the carbon fee associated with input i; p_i and Y_c are divided by the numeraire price p_{num} to maintain homogeneity of degree zero in prices; $\beta_{i,hh}$ and $\gamma_{i,hh}$ are related to the price and income elasticities of demand as described in the following section on properties of the demand system; and Γ is defined as follows:

$$\Gamma \equiv \sum_{i \in Sec} \delta_{i,hh}\left(\frac{p_i + cf_i}{p_{num}}\right)^{\beta_{i,hh}+1}\left(\frac{Y_c}{p_{num}}\right)^{\gamma_{i,hh}-1} \tag{53}$$

where the variables are as described above. The inclusion of the Γ term in the household demand equations ensures that the household budget constraint is satisfied.[40]

---

[39] $\eta_{labor,hh}$ is simply calculated as the value of labor demanded by households in the base year divided by the total value of labor supplied by households in the current year. Thus the value of labor demanded by households remains constant over time.

Household expenditures, Y_{exp}, can be defined as follows:

$$Y_{exp} = S_{hh} + p_{i=labor} \cdot x_{i=labor,hh} + \sum_{i \in Sec} (p_i + cf_i) \cdot x_{i,hh} \tag{54}$$

where S_{hh} is household savings, as described by equation (49); $p_{i=labor}$ is the price of labor; $x_{i=labor,hh}$ is the household demand for labor as given by equation (51); $x_{i,hh}$ is household demand for good i, as described in equation (52); p_i is the price of good i; and cf_i is the carbon fee associated with good i. Since the household budget constraint is satisfied, household expenditures, Y_{exp}, are equal to household disposable income, Y_{dis}, and thus the income balance condition for the household holds.

6.4.2 Properties of the Demand System

The demand system specified in equations (52) and (53) can be shown to satisfy the theoretical properties of a well-behaved demand system under certain constraints.[41] In this section, we discuss the Slutsky substitution matrix associated with the demand system specified above, and whether or not it is symmetric, negative semidefinite, and homogenous of degree zero in prices.

Elasticities can be written as a function of the β and γ parameters and the value share S_i. The own-price elasticity of demand is

$$\varepsilon_{ii} = \beta_{i,hh}(1 - S_i) - S_i \quad \forall i \in Sec \tag{55}$$

Note that the own-price elasticity of demand approaches $\beta_{i,hh}$ as the value share goes to zero.

The cross-price elasticity of demand is:

$$\varepsilon_{ij} = -S_j(\beta_{j,hh} + 1) \quad \forall i, j \in Sec \tag{56}$$

The income elasticity of demand is:

$$\varepsilon_{im} = 1 + \gamma_i - \sum_{k=1}^{n} S_k \gamma_k \quad \forall i \in Sec \tag{57}$$

---

[40] With the inclusion of Γ in the household demand function, it can be shown that the following household budget constraint holds:

$$Y_c = \sum_{i \in Sec} (p_i + cf_i) \cdot x_{i,hh}$$

where Y_c is the total value of household consumption as defined in equation (50); p_i is the price of good i; cf_i is the carbon fee associated with input i; and $x_{i,hh}$ is household demand for good i.

[41] This section assumes that the prices in equations (52) and (53) have not been normalized by the numeraire price.

However, constraints on the β and γ parameters are needed to satisfy the Slutsky symmetry conditions.

$$\beta_{i,hh} + \gamma_i = \beta_{j,hh} + \gamma_j \qquad \forall i, j \in Sec \tag{58}$$

If a constant is added to all of the gammas, it will cancel out of the demand system, so it doesn't matter what β_i and γ_i sum to, only that the sum is the same across equations. Therefore, price and income elasticities cannot be set independently of each other.

With the constraint in equation (58), the demand system (52) and (53) would satisfy the homogeneity of degree zero condition even without normalization by the numeraire price.

7. GOVERNMENT

The government sector in the model collects taxes, produces government services, makes transfer payments, and administers climate policies. The imbalance between government revenue and government expenditures results in government savings, which may be in either deficit or surplus. The various tax rates are specified exogenously, and the government deficit is also exogenous. The carbon policy may be specified as an exogenous carbon price, or in the form a quantity target that results in an endogenous carbon price. The level of government expenditure is subsequently endogenous.

7.1 Government Savings - Deficit or Surplus

Government savings, S_G, is specified exogenously to the model. Along with personal savings, corporate savings, and the net trade balance, government savings forms part of the supply side of the capital market. In the case of a government deficit, the available supply is simply reduced by the amount of the deficit.

The government's income balance condition requires that government expenditures be equal to government revenue less government savings. Since government savings is exogenous, and government revenue is determined by exogenously set tax rates, government expenditures must adjust to satisfy the government income balance condition.

7.2 Government Revenue

The government collects revenue from indirect business taxes, corporate income taxes, social security taxes, and personal income taxes, as well as any revenue from a carbon policy. Government revenue, GRv, is thus defined as follows:[42]

$$GRv = RvTx_{ibt} + RvTx_{\pi} + RvTx_{labor} + RvTx_{Y} + RvCarb \tag{59}$$

where $RvTx_{ibt}$ is revenue from the indirect business tax; $RvTx_{\pi}$ is revenue from the corporate income, or profits tax; $RvTx_Y$ is revenue from the personal income tax; $RvTx_{labor}$

[42] Other tax and subsidy instruments, such as a large set of additive and proportional taxes, are available to the model, but are generally turned off unless needed for a specific scenario.

is revenue from the proportional tax on labor, or the social security tax; and *RvCarb* is revenue from a carbon policy.

Revenue from the indirect business tax is given by:

$$RvTx_{ibt} = \sum_{j \in All} q_j \cdot p_j \cdot tx_{ibt,j} \tag{60}$$

where q_j is sector, subsector, and vintage specific output; p_j is the price of output j; and $tx_{ibt,j}$ is the indirect business tax rate for sector j.

Revenue from the corporate income tax, or profits tax, is defined as:

$$RvTx_{\pi} = \sum_{j \in All} \pi_j \cdot tx_{\pi} \tag{61}$$

where π_j is profits as defined in equation (10), and tx_{π} is the profit tax rate.

The equation below describes tax revenue from the proportional labor tax, or social security tax:

$$RvTx_{labor} = p_{i=labor} \cdot tx_{labor} \cdot q_{labor} \tag{62}$$

where $p_{i=labor}$ is the price of labor, tx_{labor} is the proportional tax rate on labor, or social security tax rate; and q_{labor} is labor supply.

Personal income tax revenue can found using the following equation:

$$RvTx_Y = \left[p_{i=labor} \cdot \left(1 - tx_{labor}\right) \cdot q_{labor} + \sum_{j \in All} \pi_j \cdot \left(1 - tx_{\pi}\right) \cdot \left(1 - s_{\pi,j}\right) \right] \cdot tx_Y \tag{63}$$

where $s_{\pi,j}$ is the rate of corporate savings or retained corporate earnings; and tx_Y is the personal income tax rate.

The revenue associated with a carbon policy, *RvCarb*, is the final source of government revenue.

$$RvCarb = \sum_{i \in Inputs} cf_i \cdot \left[x_{i,hh} + x_{i,G} + \sum_{j \in All} x_{ij_i} \right] \tag{64}$$

where $x_{i,hh}$ is household demand for input i; $x_{i,G}$ is government demand for input i; $x_{i,j}$ the demand for input i in sector, subsector, and vintage j; and cf_i is the carbon fee associated with input i. The carbon fee is essentially the price of the carbon content in one unit of the associated input, and this carbon price may be an emissions allowance price resulting form a carbon cap and trade system, or a carbon tax.

7.3 Government Expenditures

In order to satisfy the government income balance condition, government expenditures are endogenously calculated as the government revenue less government savings:

$$GEx = GRv - S_G \tag{65}$$

where GEx is government expenditures, GRv is government revenue, and S_G is government savings.

Government expenditures take the form of government final consumption, GC, and transfer payments, TR_{gov} and TR_{carb}. The government transfer payments are calculated as follows:

$$TR_{gov,t} = p_{num,t} \cdot \left[Pop_t \cdot TR_{gov,0} \cdot \left(\frac{Y_{pre,t}}{Pop_t} \right) \cdot \left(\frac{q_{labor,t}}{Pop_{WorkAge,t}} \right) \cdot Pop_{YngOld,t} \right] \tag{66}$$

where $p_{num,t}$ is the numeraire price in period t; Pop_t is the total population in period t; $TR_{gov,0}$ is the level of government transfers in the base year, which is exogenously specified; $Y_{pre,t}$ is pre-tax income in period t, which is simply disposable income from equation (48) less government transfers, and without multiplying by one minus the income tax rate; $q_{labor,t}$ is labor supply in period t, $Pop_{WorkAge,t}$ is the working age population in period t; and $Pop_{YngOld,t}$ is then non-working age population in period t.

The transfer payments associated with a carbon policy depend on what revenue recycling option is chosen. The primary revenue recycling option used here is to simply return all revenues associated with a carbon policy to households:

$$TR_{carb} = RvCarb \tag{67}$$

where $RvCarb$ is the revenue associated with the carbon policy.

Total government consumption of final goods, GC, is endogenously calculated as:

$$GC = GEx - TR_{gov} - TR_{carb} \tag{68}$$

where GEx is government expenditures, TR_{gov} are government transfers to the household sector, and TR_{carb} are government transfers to the household sector that result from a carbon policy.

Once the level of government consumption is specified, the model allocates that consumption amongst final goods, and value added. The government preferences are Leontief, so the following equation describes government demand for various goods:

$$x_{i,G} = \psi_i \cdot \frac{GC}{\sum_{i \in Var} \psi_i \cdot \left(p_i + cf_i \right)} \qquad \forall i \in Var \tag{69}$$

where $x_{i,G}$ is government final demand for input i; GC is total government consumption of final goods; ψ_i is the Leontief coefficient for government demand of input i; N is the total number of inputs; p_i is the price of input i; cf_i is the carbon fee associated with input i; and the summation in the denominator is simply a price index.

8. TRADE

Trade in the model can be handled in different ways, depending on whether the model is run with a single region or with multiple regions. Furthermore, trade can be treated differently in different sectors. Some sectors are treated as closed markets, and thus either all production is domestic, or trade is treated as an exogenous quantity. Other sectors are treated as open markets. The open markets may either be treated as having an endogenous world price, or a fixed world price.

The total value of all net exports in the economy must be equal to the negative of the deficit in the balance of payments on current account, which is part of the supply of savings in the investment market. In both single region and multiple region operation, the market for the numeraire good must be treated as an open market with a fixed price, since the numeraire price is fixed at a value of one.

8.1 Single Region Operation

When the model is run with a single region, trade may be handled in two different ways. Some markets are modeled as closed markets with exogenous net exports. Other markets are modeled as open markets with a fixed world price. In these markets net exports are endogenous.

8.1.1 Closed Markets

The treatment of trade in closed markets is quite simple. The level of net exports for a particular input (e.g. $x_{i,NetExp}$ is the level of net exports for input i) is exogenously fixed in a closed market. In these markets, the domestic price adjusts to clear the market.

8.1.2 Open Markets

Open markets are treated differently in the model. The level of net exports for input i, $x_{i,NetExp}$, in an open market is calculated as the difference between domestic production of good i, and all sources of domestic consumption of good i:

$$x_{i,NetExp} = q_i - x_{i,hh} - x_{i,G} - \sum_{j \in All} x_{ij} - \sum_{j \in SecSub} Inv_{i,j} \quad \forall i \in OpenMkt \tag{70}$$

where q_i is output of product i; $x_{i,hh}$ is household demand for product i; $x_{i,G}$ is government demand for product i; x_{ij} is demand for product i in sector, subsector, and vintage j; $Inv_{i,j}$ is the amount of product i used in the production of the capital good in sector and subsector j.

In single region operation, all open markets have a fixed exogenous world price. Since the price cannot adjust to clear the market, the level of net exports for the product,

$x_{i,NetExp}$, must endogenously adjust satisfy equation (70). At the fixed world price, consumers purchase as much as they want, and producers supply as much as desired.

Since the price of the numeraire good is exogenously set equal to one, the market associated with the numeraire good is always treated as an open market with a fixed world price. This means that net exports of the numeraire good are endogenous.

8.1.3 Single Region Trade Balance

Summing the value of net exports over all products gives the value of net exports for the economy, *NetExp*:

$$NetExp = \sum_{i \in Sec} x_{i,NetExp} \cdot p_i \tag{71}$$

where $x_{i,NetExp}$ is the level of net exports for input i; p_i is the price of good i; and N_{goods} is the number of production sectors, which is the same as the number of goods. *NetExp* must be equal to the negative of the value of the deficit in the balance of payments on current account, D, which is part of the savings supply in the investment market. The balance of payments on current account, D, is exogenously set, so verifying that net exports are indeed equal to negative D acts as a check of Walras' Law.

8.2 Multiple Region Operation

When the model is operated with multiple regions, trade can be treated in three different ways. Some markets may be closed markets, in which case they have an exogenous level of net exports. These markets are treated identically to the closed markets in single region operation. Markets may also be open, in which case they are either modeled as having a fixed world price, as in single region operation, or there may be an endogenous world price that clears a multi-region market.

8.2.1 Open Markets – Fixed World Price

In multi-region operation, markets with a fixed world price behave in a similar way to open markets in single region operation. For each region with a particular open market, equation (70) holds for that market in each region. This is restated below over the proper set of markets and regions:

$$x_{i,NetExp,l} = q_{i,l} - x_{i,hh,l} - x_{i,G,l} - \sum_{j \in All} x_{ij,l} - \sum_{j \in SecSub} Inv_{i,j,l} \qquad \begin{array}{l} \forall i \in OpenMktF \\ \forall l \in Regions \end{array} \tag{72}$$

where $q_{i,l}$ is region l's output of product i; $x_{i,hh,l}$ is household demand for product i in region l; $x_{i,G,l}$ is government demand for product i in region l; $x_{ij,l}$ is region l's demand for product i in sector, subsector, and vintage j; $Inv_{i,j,l}$ is the amount of product i used in the production of the capital good in sector and subsector j in region l.

8.2.2 Open Markets – Endogenous Price

The primary difference between single region and multiple region operation is the availability of open markets with an endogenous world price in multiple region operation. Generally when running the model, only carbon allowances are allowed to be traded in a multi-regional open market; however, the model allows for any good to be traded in such a market.

In open markets with an endogenous world price, the level of net exports for input i in each region l, $x_{i,NetExp,l}$, is calculated as the difference between domestic production of good i, and all sources of domestic consumption of good i:

$$x_{i,NetExp,l} = q_{i,l} - x_{i,hh,l} - x_{i,G,l} - \sum_{j \in All} x_{ij,l} - \sum_{j \in SecSub} Inv_{i,j,l} \quad \begin{matrix} \forall i \in OpenMktE \\ \forall l \in Regions \end{matrix} \tag{73}$$

where $q_{i,l}$ is output of product i in region l; $x_{i,hh,l}$ is household demand for product i in region l; $x_{i,G,l}$ is government demand for product i in region l; $x_{ij,l}$ is demand for product i in sector, subsector, and vintage j and in region l; $Inv_{i,j,l}$ is the amount of product i used in the production of the capital good in sector j in region l; and $OpenMkt_l$ is the set of all goods that are traded in an open market as opposed to a closed market in region l.

When the model is run with multiple regions, an endogenous world price adjusts to clear the global market for the particular good. The following equation represents the market clearing condition for an open market with an endogenous world price:

$$\sum_{l \in Regions} q_{i,l} = \sum_{l \in Regions} \left[\sum_{j \in All_l} x_{ij,l} + x_{i,hh,l} + x_{i,G,l} + \sum_{j \in Sec_l} Inv_{i,j,l} + x_{i,NetExp,l} \right] \begin{matrix} \forall i \in OpenMktE \\ \forall l \in Regions \end{matrix} \tag{74}$$

where $q_{i,l}$ is the production of good i in region l, and can be found by summing the q_j's from equation (11) over all subsectors and vintages that produce good i in a particular region; $x_{ij,l}$, described in equation (7), is the demand for input i as used in production by the sector, subsector, and vintage specific entity j in region l; All_l is set of all sector, subsector, and vintage specific production entities in region l; $x_{i,hh,l}$ is region l's household demand for good i, as described in equation (52); $x_{i,G,l}$ is demand for good i by the government of region l, as described in equation (69); $Inv_{i,j,l}$, described in equation (40), is the use of input i in the production of the capital good in sector j in region l; Sec_l is the set of all sectors, or the number of goods in region l; and $x_{i,NetExp,l}$ is net export of good i from region l, and can be found in equation (73).

Since the price of the numeraire good is exogenously set equal to one, the market associated with the numeraire good is always treated as an open market with a fixed world price. This means that net exports of the numeraire good are endogenous.

8.2.3 Multiple Region Trade Balance

Summing the value of net exports over all products gives the value of net exports for the each region, $NetExp_l$:

$$NetExp_l = \sum_{i \in Sec} x_{i,NetExp,l} \cdot p_i \quad \forall l \in Regions \tag{75}$$

where $x_{i,NetExp,l}$ is the level of net exports from region l for input i; and p_i is the price of good i. *NetExp* must be equal to the negative of the value of the deficit in the balance of payments on current account, D, which is part of the savings supply in the investment market. The balance of payments on current account, D, is exogenously set, so verifying that net exports are indeed equal to negative D acts as a check of Walras' Law.

9. EQUILIBRIUM CONDITIONS

A set of equilibrium conditions is used to close the model. These equilibrium conditions include: a set of market clearing conditions for all commodity markets; market clearing conditions for the two factor markets, labor and capital; and income balance conditions for households and government. As noted earlier in the investment section, the model does not make explicit use of the zero profits condition its set of system equations, relying instead upon the output and investment accelerators to allocate investment across sectors.

9.1 Commodity Market Clearing Conditions

The principle of no free disposability implies that the goods produced in the economy must be fully absorbed by the various consumption activities in the economy. This simply means that all of the markets must clear. For the commodity markets in the model that operate as closed markets, this means that the price must adjust so that supply is equal to demand. In open markets with a fixed price net exports of the product must adjust so all the goods produced are absorbed by the combination of domestic consumption and net exports of the particular good.

The following equation describes the set of market clearing condition for all closed commodity markets:

$$q_i = \sum_{j \in All} x_{ij} + x_{i,hh} + x_{i,G} + \sum_{j \in SecSub} Inv_{i,j} + x_{i,NetExp} \qquad \forall i \in CloseMkt \tag{76}$$

where q_i is the production of good i, and can be found by summing the q_j's from equation (11) over all subsectors and vintages that produce good i; x_{ij}, described in equation (7), is the demand for input i as used in production by the sector, subsector, and vintage specific entity j; $x_{i,hh}$ is household demand for good i, as described in equation (52); $x_{i,G}$ is government demand for good i, as described in equation (69); $Inv_{i,j}$, described in equation (40), is the use of input i in the production of the capital good in sector and subsector j; and $x_{i,NetExp}$ is net export of good i. In closed markets, $x_{i,NetExp}$ is exogenous.

In the case where goods are traded on open markets with an endogenous world price, equation (76) is modified so that all the elements are summed over all regions participating in trade, so equation (76) becomes a market clearing condition for the world commodity market.

9.2 Factor Market Clearing Conditions

There are two primary factors in the model, labor and capital. The labor market behaves in a very similar way to the commodity markets. A simple market clearing condition for labor is satisfied by adjustments in the wage rate. The market for capital is more complex. The stock of capital consists of multiple vintages of capital, and each vintage of capital is the result of the accumulation of capital from annual investment in each year within a model time step. The market clearing condition plays out in the market for current period investment, where the supply of savings and the demand for investment must balance. This balance is achieved through adjustments of the interest rate, which affects the supply of savings and the demand for investment.

The market clearing condition for the labor market is similar to that for the commodity markets:

$$q_{labor} = \sum_{j \in All} x_{i=labor,j} + x_{i=labor,hh} + x_{i=labor,G} \tag{77}$$

where q_{labor}, found in equation (46), is the quantity of labor supplied; $x_{i=labor,j}$ is the demand for labor by the sector, subsector, and vintage specific entity j, and is described in equation (7), the same equation that describes the demand for all variable inputs by firms; $x_{i=labor,hh}$ is the household demand for labor as given by equation (51); and $x_{i=labor,G}$ is the government demand for labor, which can be found in equation (69), the government demand equation for variable inputs.

The final market clearing condition is that for investment or savings:

$$\sum_{j \in Sec} I_j \cdot p_{I,j} = S_{hh} + S_G + S_{RE} - D \tag{78}$$

where I_j is investment by sector j, which for the output accelerator method used in energy-intensive sectors is given by equation (27), and the investment accelerator method used in all other industries is given by equation (19); $p_{I,j}$ is the price index for investment purchased by sector j, and is defined in equation (41); S_{hh} is household savings, as described by equation (49); S_G is government savings, which may either be in surplus or deficit, and is exogenously determined; S_{RE} is corporate savings, or retained earnings, and is described in equation (37); and D is the exogenous deficit in the balance of payments on current account. If the Walras' Law is satisfied, then the negative of D should be equal to *NetExp* from equation (71).

9.3 Income Balance

The last of the equilibrium conditions described here are the income balance conditions. Households receive income from renting out their endowments of the primary factors, labor and capital, and the factor endowments are fully employed. Households exhaust that income through expenditures on commodities, including savings. This equality between household income and household expenditures is the income balance condition for households. Similarly an income balance condition holds for government revenues and expenditures.

The income balance condition for households is satisfied if household disposable income, Y_{dis} from equation (48), is equal to household expenditures, Y_{exp} from equation (54). This condition is satisfied through a combination of equation (48), which defines Y_{dis}; and the imposition of the household budget constraint on the household demand system, which ensures that household expenditures are equal to household income. The essential equation of the model that satisfies the income balance condition is thus equation (48), which is restated below:

$$Y_{dis} = \left[w \cdot q_{labor} + \sum_{j \in All} \pi_j \cdot \left(1 - tx_\pi\right) \cdot \left(1 - s_{\pi,j}\right) \right] \left(1 - tx_Y\right) + TR_{gov} + TR_{carb} \tag{79}$$

where w is the after tax wage rate, given in equation (47); q_{labor} is household labor supply, from equation (46); π_j is profit in sector, subsector, and vintage j defined in equation (10); tx_π is the exogenous profit tax rate; $s_{\pi,j}$ is the rate of corporate savings or retained corporate earnings, from equation (36); tx_Y is the exogenous personal income tax rate; TR_{gov} are government transfers, found in equation (66); and TR_{carb} are transfers associated with a carbon policy, given in equation (67).

The income balance condition for the government given in equation (65), and is restated here:

$$GEx = GRv - S_G \tag{80}$$

where GEx is government expenditures; GRv is government revenue as defined in equation (59); and S_G is government savings, which may either be in deficit or surplus, and is specified exogenously.

9.4 Basic Model Tests

There are a few additional checks that can be preformed on the model to test for accounting leaks, or basic programming errors. These include a check of Walras Law, which involve verifying that net exports are equal to negative of the deficit in the balance of payments on current account; and a check of the homogeneity properties of the model.

9.4.1 Numeraire and Homogeneity

The model does not endogenously determine the absolute price level. Therefore, one price must be set exogenously, and this price is known as the numeraire. The numeraire good in this model is the good produced by the everything else sector, so the price of the everything else sector is the numeraire price, p_{num}. Since the absolute price level is not endogenously determined, the model should be homogeneous of degree one in prices and values. That is if the numeraire price, and all other exogenous prices and values, are doubled, then all endogenous prices and values will double, and all quantities will remain unchanged.

9.4.2 Walras' Law

According to Walras' Law, if *n-1* markets clear, then the n^{th} market must also clear. This means that one equation must be dropped, since only *n-1* of the equations are independent. The equation that is dropped from the model is the condition that net exports, *NetExp*, are equal to the negative of the deficit in the balance of payments on current account, D.[43] In order to verify that Walras' Law is satisfied, we create a new variable, *Walras*, and check that it is equal to zero:

$$Walras = NetExp + D \tag{81}$$

The deficit in the balance of payments on current account is exogenously set in the model. The net exports from any particular sector may be exogenously set if the trade for that sector takes place in a closed market, or endogenously determined if trade in that sector is modeled as an open market. The everything else sector, which is the sector of the numeraire good, is always modeled as an open sector with a fixed price, so even if all other sectors are closed, the total level of net exports, *NetExp*, will be endogenous. If the Walras' Law is satisfied, then negative *D* should be equal to *NetExp*, and thus *Walras* will be equal to zero.

9.5 Counting Equations and Endogenous Variables

With the full set of equilibrium conditions specified, it is now possible to count then number of equations and endogenous variables to verify that the model is not over or under determined. Table A.7 at the end of this document lists all the endogenous variables in the model along with all the corresponding equations in the model. The sets over which each endogenous variable and equation are defined are given, so the table shows that the total number of endogenous variables is equal to the total number of equations.

It is also possible to condense the set of equations and variables reduce the number that need to be accounted for. The condensed set of equations and unknowns consist of the following for the model base case: one market clearing condition for each element of the set *CloseMkt*, along with one endogenous price p_i for each element of the set *CloseMkt*; one market clearing condition for the labor market, and one endogenous wage rate; one market clearing condition for investment or savings, with endogenous interest rate; an endogenous variable for household income, along with an equation setting household income equal to the sources of income in order to satisfy the household income balance condition; an endogenous variable for government expenditures, along with an equation setting government expenditures equal to the sources of revenue in order to satisfy the government income balance condition; and finally, in policy run with an emissions limit, an endogenous carbon price, along with an equation clearing the market for carbon allowances.

10. GREENHOUSE GAS EMISSIONS

Emissions of the following suite of gases are currently tracked in the model.

- CO_2 emissions from fossil fuel combustion processes.

[43] One might expect that the market clearing condition for the numeraire good would be dropped to satisfy Walras' Law. However, since the market for the numeraire good is treated as an open market with an endogenous level of net exports, there is not a market clearing condition associated with the market for the numeraire good that can be dropped. Thus the approach described in this section is used instead.

- CH_4 emissions which emanate from the production and distribution of natural gas, mining of coal, from the raising of ruminant animals, the growing of rice, from sanitary landfills, and from combustion processes (principally biomass burning).
- N_2O emissions from combustion processes, fertilizer use, selected natural sources.
- HFC-23 emissions from the production of HCFC-22.
- Short lived HFC emissions from various uses as substitutes for ozone-depleting substances, including losses from refrigeration and air-conditioning equipment, foam blowing, aerosol propellants, cleaning solvents, and fire extinguishers.
- PFC emissions from aluminum and semiconductor production.
- SF_6 emissions from use in electrical switch gear and as a cover gas in magnesium smelting.

In the absence of a carbon policy, emissions are released into the atmosphere without cost, and thus have no effect on either the cost of production or the optimum demand for investment goods. In general, the release of CO_2 to the atmosphere is considered to be proportional to the energy content of the specific fuel by a fixed ratio and therefore largely independent of the sector or subsector in which the fossil fuel form is combusted. In contrast, since emissions of the non-CO_2 greenhouse gases are not limited to fuel use activities, a different procedure is required for tracking these emissions.

While greenhouse gas emissions in the model are calculated in different ways for CO_2 and non-CO_2 gases, the same general strategy is used for all emissions. Emissions are assumed to be proportional to the scale of input utilization. The model uses a set of emissions sources that are each associated with the release of a particular greenhouse gas in the model. Each source is also associated with a production sector in the model that is related to the actual emissions activity. An emissions factor for each source links the quantity of output in the associated production sector with actual emissions. Here is a table showing the emissions sources in the model along with their associated gases and sectors.

Table 10.1 Greenhouse Gas Emissions Sources in the SGM

Gas	Emissions Source	Associated Production Sector
CO_2	Oil Combustion	Crude oil production
	Gas Combustion	Natural gas production
	Coal Combustion	Coal Production
CH_4	Coal Production	Coal Production
	Enteric	Other agriculture
	Natural Gas Systems	Distributed gas
	Oil Systems	Crude oil production
	Landfills	Everything else
	Manure	Other agriculture
	Other Agricultural Methane	Other agriculture
	Other Non-Agricultural Methane	Everything else
	Wastewater	Everything else
HFC-23	HFC-23	Everything else
HFCs	Ozone Depleting Substances Substitutes	Everything else
N_2O	Industrial Processes	Everything else
	Manure	Other agriculture
	Mobile Source	Everything else
	Soil	Other agriculture
	Stationary Source	Everything else
PFCs	Aluminum	Everything else
	Semiconductor	Everything else
SF_6	Electricity Distribution	Electricity generation
	Magnesium	Everything else

Note that since the model contains a large amount of detail on energy production and transformation, and since CO_2 emissions are linked to fuel use activities, CO_2 emissions sources are closely related to their associated production sectors in the model. Emissions of the non-CO_2 gases originate from a much broader set of sources, therefore the production sectors they are associated with in the model are not as closely related to the source. The specifics of calculating emissions in the model will be discussed in sections below.

10.1 CO_2 Emissions

The combustion of fossil fuels drives CO_2 emissions in the model. Essentially, the model tracks the domestic demand for fossil fuels in energy units, and converts this to emissions using an emissions factor relating the energy content of the fuel to the carbon content. The following equation is used to find the carbon equivalent emissions of CO_2 from oil combustion in a particular region:

$$Em_{s=OilC} = ef_{s=OilC} \cdot \left[(1-\kappa_{Oil}) \cdot \left(q_{j=Oil} - x_{i=Oil,NetExp} \right) \cdot ej_{j=Oil} - x_{i=RefOil,NetExp} \cdot ej_{j=RefOil} \right] \quad (82)$$

where $Em_{s=OilC}$ is the carbon equivalent emissions of CO_2 from oil combustion, measured in millions of metric tons of carbon equivalent emissions (MtCe); $ef_{s=OilC}$ is the emissions factor for oil combustion, or the oil-specific carbon content in MtCe per exajoule (EJ) of the energy source (MtCe/EJ is equivalent to kgCe/GJ); $q_{j=oil}$ is domestic production of oil; $x_{i=Oil,NetExp}$ is net exports of oil; so, domestic production of oil less net exports of oil is a measure of the total domestic demand for goods from the oil production sector; $ej_{j=oil}$ is a conversion factor from output in dollars to output in exajoules for the oil sector; κ_{Oil} is the percentage of output from the oil production sector that goes to non-energy uses, this accounts for the fact that not all oil products are combusted; $ej_{j=RefOil}$ is a conversion factor from output in dollars to output in exajoules for the refined oil sector; and $x_{i=RefOil,NetExp}$ is net exports of refined oil products, which ensures that emissions from the combustion of refined oil products are attributed to the country where they are combusted. The entire term in brackets represents the amount of oil that is combusted domestically in exajoules, so emissions are found by multiplying this term by the emissions factor.

Emissions from the natural gas combustion and coal combustion sources are similarly calculated. Carbon equivalent CO2 emissions from natural gas combustion are found in a particular region using the equation below:

$$Em_{s=GasC} = ef_{s=GasC} \cdot \left[(1-\kappa_{Gas}) \cdot \left(q_{j=Gas} - x_{i=Gas,NetExp} \right) \cdot ej_{j=Gas} - x_{i=DGas,NetExp} \cdot ej_{j=DGas} \right] \quad (83)$$

where $Em_{s=GasC}$ is the carbon equivalent emissions of CO_2 (MtCe) from natural gas combustion; $ef_{s=GasC}$ is the emissions factor (MtCe/EJ) for natural gas combustion; total domestic demand for goods from the natural gas production sector is equal to $q_{j=Gas}$, domestic production of natural gas, less $x_{i=Gas,NetExp}$, net exports of natural gas; $ej_{j=Gas}$ is the conversion factor from dollars to exajoules for the natural gas sector; the percentage of output from the gas production sector that goes to non-energy uses is κ_{Gas}; and $x_{i=DGas,NetExp}$ accounts for the trade of distributed natural gas, with $ej_{j=DGas}$ is the conversion factor from dollars to exajoules for distributed natural gas.

Finally the carbon equivalent CO_2 emissions from coal combustion are given by the following equation:

$$Em_{s=CoalC} = ef_{s=CoalC} \cdot \left[(1-\kappa_{Coal}) \cdot \left(q_{j=Coal} - x_{i=Coal,NetExp} \right) \cdot ej_{j=coal} \right] \quad (84)$$

where the carbon equivalent emissions of CO_2 (MtCe) from coal combustion is represented by Em_{CoalC}; the emissions factor (MtCe/EJ) for coal combustion is ef_{CoalC}; the total domestic demand for goods from the coal production sector is $q_{j=Coal}$, domestic coal production, less $x_{i=Coal,NetExp}$, net exports coal; $ej_{j=coal}$ is the conversion factor from dollars to exajoules for the coal sector; and κ_{Coal} is the percentage of output from the coal production sector that goes to non-energy uses.

In the presence of a carbon policy, CO_2 emissions reductions require reductions in the amount of fossil fuel combusted. In the model this means reducing the emissions activities associated with the release of CO_2, which requires reducing the domestic demand for coal, oil, and natural gas production.[44]

10.2 Non-CO_2 Emissions

For non-CO_2 greenhouse gases, emissions factors do not represent a stoichiometric relationship between the output and actual emissions. In some cases this is because the relationship between emissions and the actual emissions activity is not stoichiometric, in others it is because the actual emissions activity is much more narrowly defined than the production sector the emissions are associated with in the model. Instead, base year emissions for each source, $Em_{s,1990}$ measured in MtCe[45], are used as an input, and the emissions factors, ef_s measured in MtCe per dollar of output, are calculated by dividing base year emissions, $Em_{s,t=0}$, by the total domestic demand from the associated production sector:

$$ef_s = \frac{Em_{s,t=0}}{\left(q_{j,t=0} - x_{j,NetExp,t=0}\right)\cdot ej_j} \qquad \forall s \in NonCO_2 \tag{85}$$

where $q_{j,t=0}$ is the base year domestic production, measured in dollars, for sector j, which is the production sector associated with emissions source s; $x_{j,NetExp,t=0}$ is base year net exports from sector j; and ej_j is the conversion factor form dollars to exajoules for sector j.[46] Emissions in future time periods, $Em_{s,t}$, are calculated as follows:

$$Em_{s,t} = ef_s \cdot \left(1 + efadj_{s,t}\right)\cdot\left(q_{j,t} - x_{j,NetExp,t}\right)\cdot ej_j \qquad \forall s \in NonCO_2 \tag{86}$$

where $efadj_{s,t}$ is a time dependent adjustment parameter for the emissions factor for source i, this is used to tune emissions to an exogenous emissions forecast.

Emissions reductions for non-CO_2 gases are accomplished in two ways in the model. As is the case with CO_2 emissions, emissions of non-CO_2 gases can be reduced by lowering output from the associated production sectors. The second mechanism for non-CO_2 emissions reductions are exogenous marginal abatement cost curves. The following equation shows how emissions of non-CO_2 gases are calculated when using a marginal abatement cost curve.

$$Em_{s,t} = ef_s \cdot \left(1 + efadj_{s,t}\right)\cdot\left(1 - mac_{s,t}\right)\cdot\left(q_{j,t} - x_{j,NetExp,t}\right)\cdot ej_j \qquad \forall s \in NonCO_2 \tag{87}$$

---

[44] Another option for reducing CO_2 emissions is capture and storage. When electricity generating technologies with carbon capture and storage are used in the model, a percentage of the fuel used is subtracted from the domestic consumption of that fuel in the emissions calculation. The percentage is equal to the percentage of emissions captured by the technology.

[45] Emissions of non-CO_2 gases are converted to carbon equivalent emissions using global warming potentials from the 1996 IPCC Second Assessment Report.

[46] For sectors that are associated with energy production or conversion, ej_i is calculated as the quotient of the sectors output in units of energy from the energy balance tables and the sector output in dollars from the input output tables. For all other sectors ej_i is equal to one

where $mac_{s,t}$ is the percentage reduction in emissions that can be achieved, given a particular carbon price, for emissions from source *s* in time period *t*. In the presence of a carbon policy that applies to non-CO_2 greenhouse gases, emissions from any particular source will be reduced as output from the associated sector falls, and emissions will be further reduced by an amount indicated by the marginal abatement cost curve for that source at the prevailing carbon price.

10.3 Carbon Price

The additive carbon fee associated with a particular input relates the carbon price to the greenhouse gas emissions associated with a particular input. The following equation is used to calculate the additive carbon fee, $cf_{i,t}$:

$$cf_{i,t} = p_{carbon,t} \cdot \sum_{s \in SecSource_i} ef_s \cdot \left(1 + efadj_{s,t}\right) \cdot \left(1 - mac_{s,t}\right) \cdot ej_j \cdot GWP_s \qquad \forall i \in Var \tag{88}$$

where p_{carbon} is the price of carbon; ef_s is the emissions factor for greenhouse gas source *s*; $efadj_s$ is the time dependent adjustment parameter, which is equal to zero for CO_2 sources; $mac_{s,t}$ is the exogenous marginal abatement parameter for non-CO_2 emissions; ej_j is the conversion factor from dollars to exajoules for sector *j*, which is the sector associated with emissions source *s*; and GWP_s is the global warming potential for gas who's emissions are associated with source *s*, this converts the units from the weight of each particular gas, to carbon equivalents. The summation is over all greenhouse gas emissions sources associated with a particular sector *i*.

The carbon price, p_{carbon}, may either be endogenous or exogenous depending on the carbon policy being modeled. With any policy that specifies a carbon price, p_{carbon} is simply set exogenously. For policies that instead specify an emissions target, p_{carbon} is an endogenous variable, and an additional equation is needed for the carbon market which is cleared by the carbon price.

$$\sum_{s \in Sources} Em_{s,t} \cdot GWP_s = EmLimit_t \tag{89}$$

where $EM_{s,t}$ is emissions from source *s* in period *t*; GWP_s is the global warming potential for gas who's emissions are associated with source *s*; and $EmLimit_t$ is the exogenous emissions limit in period *t*.

Table A.1 List of Endogenous Variables Used

Variable	Description
$a_{N,jj}$	The capital output ratio for subsector *jj*. Defined $\forall jj \in Sub_j$, $\forall j \in Sec$.
$cfac_{i,j}$	Factor used to convert current carbon fee to present value of a discrete stream of future carbon fees associated with input *i* in sector *j*. Defined $\forall i \in Var$, $\forall j \in Sec$.
cf_i	Carbon fee associated with input *i*. Defined $\forall i \in Var$.
C_{jj}	Variable logit shares are based upon (may be levelized cost). Defined $\forall jj \in Sub_j$, $\forall j \in Sec$.
Em_s	Emissions from source *s*. Defined $\forall s \in Sources$.
$e\pi_{j,t}$	Expected future profits for sector and subsector *j* in period *t*. Defined $\forall j \in SecSub$.
$e\pi rate_{j,t}$	Expected future profit rate for sector *j* in period *t*. Defined $\forall j \in Sec$.
$e\pi Sec_{j,t}$	Expected profits for sector *j* in period *t*. Defined $\forall j \in Sec$.
$fac_{i,j}$	Factor used to convert current prices to present value of a discrete stream of future prices for product *i* in sector *j*. Defined $\forall i \in Var$, $\forall j \in Sec$.
$fac_{N,j}$	Factor used in converting purchase price of capital to rental price. Defined $\forall j \in Sec$.
Γ	Defined in equation (53).
GC	Government consumption.
GEx	Government expenditures.
GRv	Total government revenue.
$I_{j,t}$	Investment by sector *j* in period *t*. Defined $\forall j \in Sec$.
$\tilde{I}_{j,t}$	Estimated investment by sector *j* in period *t*. Defined $\forall j \in OutAcc$.
$I_{jj,t}$	Subsector level investment in period *t*.
$Inv_{i,j,t}$	Investment by sector *j* spent on input *i* in period *t*. Defined $\forall i \in Sec$, $\forall j \in Sec$.
$NetExp$	The value of net exports for the economy.
p_{carbon}	Carbon price, may be endogenous or exogenous depending on the carbon policy in use.
p_i	Price of input *i*. Defined $\forall i \in Var$, endogenous $\forall i \in CloseMkt$ and for *i=labor*.
$p_{I,j}$	Purchase price of capital goods in sector and subsector *j*. Defined $\forall j \in Sec$.
π_j	Profits in sector *j*. Defined $\forall j \in All$.
$p_{N,j}$	Rental price of capital in sector *j*. Defined $\forall j \in Sec$.
q_j	Quantity of output for sector *j*. Defined $\forall j \in All$.
$\tilde{q}_{j,t}$	Estimated gross output from sector *j* in period *t*. Defined $\forall j \in OutAcc$.
$\tilde{q}_{j,t,v=new}$	Estimated output from the new vintage of capital in sector *j* in period *t*. Defined $\forall j \in OutAcc$.
$\tilde{q}_{j,t,v=old}$	Estimated output from old vintages of capital in sector *j* in period *t*. Defined $\forall j \in OutAcc$.
q_{labor}	Household labor supply.
$r_{j,t}$	Interest rate in sector *j* during period *t*. Defined $\forall j \in Sec$.
r_t	Economy wide interest rate during period *t*.
$RvCarb$	Revenue from a carbon policy.
$RvTx_{ibt}$	Revenue from the indirect business tax.
$RvTx_{labor}$	Revenue from the proportional tax on labor, or social security tax.
$RvTx_{\pi}$	Revenue from the corporate income or profits tax.
$RvTx_Y$	Revenue from the personal income tax.

$share_{jj}$	Logit share for subsector *jj*. Defined $\forall jj \in Sub_j$, $\forall j \in Sec$.
S_{hh}	Household savings.
$s_{\pi,j}$	The rate of corporate savings or retained corporate earnings in sector, subsector, and vintage *j*. Defined $\forall j \in All$.
S_{RE}	Total corporate savings, or retained earnings.
TR_{carb}	Government transfer payments associated with a carbon policy.
TR_{gov}	Government transfer payments to households.
V_j	Defined in equation (8). Defined $\forall j \in All$.
w	After tax wage rate.
$Walras$	Variable used to test Walras' Law. Should be equal to zero if Walras' Law is satisfied.
$x_{i,G}$	Government final demand for input *i*. Defined $\forall i \in Var$.
$x_{i,hh}$	Household demand for input *i*. Defined $\forall i \in Inputs$.
$x_{i,NetExp}$	The level of net exports for input *i*. Defined $\forall i \in Sec$, endogenous $\forall i \in OpenMkt$.
x_{ij}	Sector j's factor demand for input *i*. Defined $\forall i \in Var$, $\forall j \in All$.
$x_{N,jj,v=new}$	Actual amount of the fixed input, capital, demanded by the newest vintage of production, *v=new*, of sector and subsector *j*. Defined $\forall jj \in SecSub$.
$x_{N,jj,v\neq new}$	Actual amount of the fixed input, capital, demanded by the old vintages of production, $v \neq new$, of sector and subsector *j*. Defined $\forall jj \in SecSub$. (This is simply carried over from the previous period, with a rule for shutting down vintages that are no longer profitable. An explicit equation is not given above for this variable).
$\tilde{x}_{N,j,v=new}$	Estimated amount of capital (input *N*) demanded by the newest vintage of production, *v=new*, of sector *j*. Defined $\forall j \in OutAcc$.
Y_c	Total value of household consumption.
Y_{dis}	Household disposable income.
Y_{exp}	Household expenditures
Y_{pre}	Pre-tax household income.
$Ze_{j,t}$	Defined in equation (17). Defined $\forall j \in SecSub$.
Z_j	Defined in equation (9). Defined $\forall j \in All$.

Table A.2 Specific Instances and Alternate Notation for Endogenous Variables

Variable	Description
cf_k	Carbon fee associated with input k (alternate notation for cf_i).
C_k	Variable logit shares are based upon (may be levelized cost). Alternate notation for C_{jj}.
$Em_{s=CoalC}$	Emissions from coal combustion, specific instance of Em_s.
$Em_{s=GasC}$	Emissions from gas combustion, specific instance of Em_s.
$Em_{s=OilC}$	Emissions from oil combustion, specific instance of Em_s.
$p_{i=labor}$	Price of labor. Specific instance of p_j.
p_j	Price of output j. There is a single price for each output, so p_j is simply a restatement of p_i
p_k	Price of input k (alternate notation for p_i when multiple two indexes are needed in one equation)
p_{num}	Price of the numeraire good. Specific instance of p_j.
$x_{i=labor,G}$	Government demand for the input labor, specific instance of $x_{i,hh}$.
$x_{i=labor,j}$	Sector j's demand for the input labor, specific instance of $x_{i,hh}$.
$x_{i=labor,hh}$	Household demand for the input labor, specific instance of $x_{i,hh}$.
x_{kj}	Sector j's factor demand for input k (alternate notation for x_{ij})
$x_{N,j,v=new}$	Actual amount of the fixed input, capital, demanded by the newest vintage of production, $v=new$, of sector j. Illustrative, actual calculation is done at subsector level ($x_{N,jj,v=new}$).
x_{Nj}	Sector, subsector and vintage j's factor demand for input N (capital). Alternate notation for $x_{N,j,v}$.

Table A.3 List of Exogenous Variables Used

Variable	Description
D	The balance of payments on current account.
$EmLimit_t$	Exogenous emissions limit in period t.
GWP_s	Global warming potential for the gas associated with emissions source s.
N	Number of inputs to production.
$NStep$	Number of years in a model time step
ω_j	Sector specific adder to the market interest rate. Defined $\forall j \in Sec$.
Pop_t	Population in period t.
$Pop_{WorkAge,t}$	Working age population in period t.
$Pop_{YngOld,t}$	Non-working age population in period t.
S_G	Government savings (deficit or surplus).
T_j	Capital lifetime in sector j measured in years.
$tx_{ibt,j}$	Indirect business tax on sector j.
tx_{labor}	Proportional tax on labor, or social security tax.
tx_{π}	Corporate income, or profits tax.
tx_Y	Personal income tax.

Table A.4 List of Sets Used

Set	Description
Inputs	A set of all inputs to production.
Var	A set of all variable inputs to production.
Sec	A set of all the sectors, or all the types of goods or commodities.
SecSub	A set of all sectors and subsectors, or all ways of producing goods.
All	A set of all sectors, subsectors, and vintages, or a set of all profit maximizing entities.
Sub_j	A set of all the subsectors within a particular sector $\forall j \in Sec$.
OutAcc	A set of all sectors that use the investment accelerator method.
InvAcc	A set of all sectors that use the output accelerator method.
OpenMkt	A set of all sectors whose goods are traded in an open market.
CloseMkt	A set of all sectors whose goods are traded in a closed market.
Regions	A set of all regions in the model.
OpenMktF	A set of all open markets with a fixed world price. Used in multi-region operation.
OpenMktE	A set of all open markets with an endogenous world price. Used in multi-region operation.
Sources	A set of all sources of greenhouse gas emissions.
$NonCO_2$	A set of all sources of non-CO_2 greenhouse gas emissions.
$SecSource_i$	A set of all greenhouse gas emissions sources associated with a particular sector $i \in Sec$.

Table A.5 List of Parameters Used

Parameter	Description
Production function parameters	
α_{oj}	CES production function parameter.
α_{ij}	CES production function parameter (note that α_{kj} is an alternate notation for α_{ij}).
σ	The elasticity of substitution.
ρ	Equal to $(\sigma-1)/\sigma$.
α_{Nj}	Specific instance of CES production function parameter α_{ij}.
Price expectations parameters	
$pc_{i,yr}$	The percentage change in the price of product *i* in year *yr* from the current period price.
$cfc_{i,yr}$	The percentage change in the carbon fee for input *i* in year *yr* from the current period carbon fee.
Investment accelerator parameters	
$base\_rate$	Parameter used to represent capital deepening, or an overall increase in the amount of capital per worker.
$sclinv$	An exogenous scalar multiplier.
Logit share parameters	
b_{jj}	Calibration parameter in the logit share equation. (b_k is an alternate notation for b_{jj})
λ	Determines the rate one technology can substitute for another.
Retained earnings parameters	
$remax_j$	Represents the maximum rate of retained earnings in sector, subsector, and vintage *j*.
δre_j	Scale parameter for retained earnings.
ϕre_j	Sector specific parameter that determines the sensitivity of corporate savings to the interest rate *r*.
Production of capital goods parameters	
$CapMat_{ij}$	An exogenously determined matrix of Leontief coefficients.
Technical change parameters	
$\gamma_{0,j,s}$	parameter that represents the rate of neutral technical change for sector *j* in period *s*.
$\gamma_{i,j,s}$	parameter that represents the rate of technical change for the use of input *i* in sector *j* during period *s*.
Vintages and the elasticity of substitution related parameters	
αold_{ij}	Production function coefficient for old vintages. (*old* suppressed elsewhere)
αnew_{ij}	Production function coefficient for new vintages. (*new* suppressed elsewhere)
αold_{0j}	Production function coefficient for old vintages. (*old* suppressed elsewhere)
αnew_{0j}	Production function coefficient for new vintages. (*new* suppressed elsewhere)
σ^*	The short-run elasticity of substitution used for old vintages. (σ is the long-run elasticity of substitution used for new vintages.)
ρ^*	Equal to $(\sigma^*-1)/\sigma^*$.
Household labor supply parameters	
θ_{labor}	Maximum potential share of working age population employed in any given year.
δ_{labor}	A scale parameter for labor supply.
ϕ_{labor}	Labor supply responsiveness coefficient.
Household savings parameters	
θ_{hhsave}	Maximum potential savings rate.
δ_{hhsave}	A scale parameter for household savings.
ϕ_{hhsave}	Determines the sensitivity of households to the interest rate *r*.
Household demand parameters	

$\eta_{labor,hh}$	Household labor demand intensity factor.
$\delta_{i,hh}$	Household demand intensity factor for good *i*.
$\beta_{i,hh}$	Price elasticity of demand by households for good *i*.
$\gamma_{i,hh}$	Income elasticity of demand by households for good *i*.
Government expenditures parameters	
ψ_i	Leontief coefficient for government demand of input *i*.
Emissions parameters	
ef_s	Emissions factor for emissions source *s*.
κ_{Oil}	The percentage of output from the oil production sector that goes to non-energy uses.
κ_{Gas}	The percentage of output from the natural gas production sector that goes to non-energy uses.
κ_{Coal}	The percentage of output from the coal production sector that goes to non-energy uses.
$ej_{j=Oil}$	A conversion factor from output in dollars to output in exajoules for the oil production sector.
$ej_{j=Gas}$	A conversion factor from output in dollars to output in exajoules for the natural gas production sector.
$ej_{j=Coal}$	A conversion factor from output in dollars to output in exajoules for the coal production sector.
ej_j	A conversion factor from output in dollars to output in exajoules for sector *j*.
$efadj_{s,t}$	A time dependent adjustment parameter for the emissions factor for source *i*, this is used to tune emissions to an exogenous emissions forecast.
$mac_{s,t}$	The percentage reduction in emissions that can be achieved, given a particular carbon price, for emissions from non-CO_2 source *s* in time period *t*.

Table A.6 List of Subscripts Used

Subscript	Description
Indexes	
j	Index over elements of *All, Sec, InvAcc, OutAcc,* or *SecSub.*
i	Index over elements of *Inputs, Var* .
k	Index over elements of *Var,* or Sub_j.
jj	Index over elements of Sub_j.
t	Index over five year model periods (note that this subscript is often suppressed).
yr	Index over individual years.
l	Index over elements of *Regions.*
s	Index over elements of *Sources,* $NonCO_2$, or an index over five year model periods.
v	Index over vintages of capital, v=*new* indicates newest vintage, v=*old* indicates sum over all old vintages.
Specific instances of an index	
num	Numeraire, generally used as a subscript of price to indicate the numeraire price.
G	Government.
hh	Household.
$NetExp$	Net exports.
RE	Retained earnings
I	Used as a subscript on price to indicate price of investment.
N	When used as a subscript, denotes the N^{th} input, which is the fixed factor capital.
$WorkAge$	Working age – type of population.
$YngOld$	Young and Old, or non-working age – type of population.
Government related	
Y	Subscript used to denote income tax.
$labor$	Subscript used to denote labor or social security tax.
π	Subscript used to denote profit tax.
ibt	Subscript used to denote indirect business tax.
gov	Subscript used to indicate government transfers.
$carb$	Subscript used to indicate government transfers from carbon policy revenue.
Emissions related	
$s=OilC$	Oil combustion emissions source.
$s=GasC$	Natural Gas combustion emissions source.
$s=CoalC$	Coal combustion emissions source.
$j=Oil$	Oil production sector.
$j=RefOil$	Refined oil sector.
$j=Gas$	Natural gas production sector.
$j=DGas$	Distributed natural gas sector.
$j=Coal$	Coal production sector.
$carbon$	Used as a subscript on price to indicate the carbon price

Table A.7 Counting Equations and Endogenous Variables

Endogenous Variable	Corresponding Equations	Sets Variables and Equations are Defined Over: *1*	*Var*	*Sec*	*SecSub*	*All*	*OutAcc*	*InvAcc*	*OpenMkt*	*CloseMkt*	*Sources*
$a_{N,jj}$	(32)				x						
$cfac_{i,j}$	(15)		x	x							
cf_i	(88)		x								
C_{jj}	(30)				x						
Em_s	(82), (83), (84), (86)										x
$e\pi_{j,t}$	(16)				x						
$e\pi rate_{j,t}$	(18)			x							
$e\pi Sec_{j,t}$	(31)			x							
$fac_{i,j}$	(13)		x	x							
$fac_{N,j}$	(25)			x							
Γ	(53)	x									
GC	(68)	x									
GEx	(65) or (80)	x									
GRv	(59)	x									
$I_{j,t}$	(19), (27)						x	x			
$\tilde{I}_{j,t}$	(26)						x				
$I_{jj,t}$	(34)				x						
$Inv_{i,j,t}$	(40)			x2							
$NetExp$	(71)	x									
p_{carbon}	(89)	x									
p_i	(76), (77)	x								x	
$p_{I,j}$	(41)			x							
π_j	(10)					x					
$p_{N,j}$	(24)			x							
q_i	(11)					x					
$\tilde{q}_{j,t}$	(21)						x				
$\tilde{q}_{j,t,v=new}$	(22)						x				
$\tilde{q}_{j,t,v=old}$	Equation not described above						x				
q_{labor}	(46)	x									
$r_{j,t}$	(14)			x							
r_t	(78)	x									
$RvCarb$	(64)	x									
$RvTx_{ibt}$	(60)	x									

$RvTx_{labor}$	(62)	x									
$RvTx_{\pi}$	(61)	x									
$RvTx_{Y}$	(63)	x									
$share_{jj}$	(29)				x						
S_{hh}	(49)	x									
$s_{\pi,j}$	(36)					x					
S_{RE}	(37)	x									
TR_{carb}	(67)	x									
TR_{gov}	(66)	x									
V_j	(8)					x					
w	(3) or (47)	x									
$Walras$	(81)	x									
$x_{i,G}$	(69)		x								
$x_{i,hh}$	(51), (52)		x								
$x_{i,NetExp}$	(70)								x		
x_{ij}	(7)		x			x					
$x_{N,jj,v=new}$	(35)				x						
$x_{N,jj,v\neq new}$	Equation not described above	*			x						
$\tilde{x}_{N,j,v=new}$	(33)						x				
Y_c	(50)						x				
Y_{dis}	(48) or (79)	x									
Y_{exp}	(54)	x									
Y_{pre}	Defined in text, in reference to equation (66)	x									
$Ze_{j,t}$	(17)					x					
Z_j	(9)					x					

* for all sectors and subsectors, $x_{N,jj,v\neq new}$ is defined for all old vintages.

REFERENCES

Brenkert, A.L., R.D. Sands, S.H. Kim, H.M. Pitcher. 2004. Model Documentation: The Second Generation Model, http://www.epa.gov/air/pdfs/SGMdocumentation.pdf

Clarke, J.F. and J.A. Edmonds. 1993. Modeling Energy Technologies in a Competitive Market, *Energy Economics* 15(2):123-129.

Edmonds, J.A., H.M. Pitcher, D. Barns, R. Baron and M.A. Wise. 1993. Modeling Future Greenhouse Gas Emissions: The Second Generation Model Description, in *Modeling Global Change*, Lawrence R. Klein and Fu-chen Lo, eds. United Nations University Press, New York.

Edmonds, J.A., H.M. Pitcher, R.D. Sands. 2004. Second Generation Model 2004: An Overview, http://www.epa.gov/air/pdfs/SGMoverview.pdf

Fawcett, A.A. and R.D. Sands. 2005. Non-CO_2 Greenhouse Gases in the Second Generation Model, *Energy Journal* (submitted).

Sands, R.D. and A.A. Fawcett 2005. The Second Generation Model: Data, Parameters, and Implementation. Pacific Northwest National Laboratory, PNNL-15431.